The inverse square law and the 3-dimensional world

In an earlier article, I mentioned that Newton showed that the Sun's gravitational attractions to planets are inversely proportional to the distance to the planet. In this article, I explain why.

Suppose there is something called a "gravitational field." A gravitational field emanates from a massive body. See Fig. 1. The heavier the massive body, the stronger the gravitation field. From this field picture, it is apparent that the farther the distance to the object, the less the gravitational force, since the gravitational field is less dense.

This is a rough, qualitative statement, but actually one can say this more precisely and quantitatively. The strength of the gravitational force is proportional to how "many" gravitational fields pass through a unit area. Suppose you draw a sphere with radius 1 meter around a massive object, and let's say that "six" gravitational fields come out from it. See Fig. 2. Then, it is easy to see that $6/(4\pi)$ numbers of gravitational fields pass through per unit area, in square meters, since the area of the sphere with radius 1 meter is 4π square meters. This density of a gravitational field is something that is proportional to the gravitational force at a distance of 1 meter from the object. Now, how much weaker will the gravitational force be if the distance is 2 meters? To find this out, let's draw a sphere with radius 2 meters around the object. See Fig. 3. The number of gravitational fields passing through the sphere is still same, but the area of the sphere is quadrupled; we know that the area of a sphere is proportional to the square of radius. Therefore we conclude that the gravitational force is quartered if the distance is doubled, as the gravitational force is proportional to the number of gravitational field passing through per unit area in square meters. Similarly, if the distance is tripled, the gravitational force would be one-ninth as strong.

This is the reason why the gravitational force is inversely proportional to the distance squared. Actually, electric force, called "Coulomb force," is also inversely proportional to the distance squared for the same reason. In these two cases, we had to introduce the new concept, "field" to better understand the inverse square law, but there is another case where understanding of the inverse square law is much more intuitive. In an earlier article on history of astronomy, we introduced the concept of absolute brightness of star and apparent brightness of star; if the absolute brightness of star is same, the farther a star is the smaller its apparent brightness. Let's make this quantitatively. Think of Fig. 2 and Fig. 3 again, but now located at the center is a star emitting light. Let's say it emits 100 Joules of energy per a second. Then, at 1 meter away (Fig. 2), the amount of light energy per unit area one receives will be $100/(4\pi)$ Joules per meter squared per second. Of course, this will be quartered at 2 meter away, and one-ninth at 3 meter away.

Fig. 1

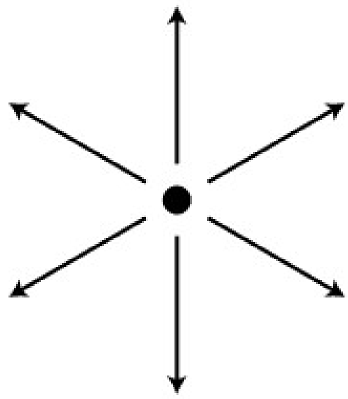


Fig. 2

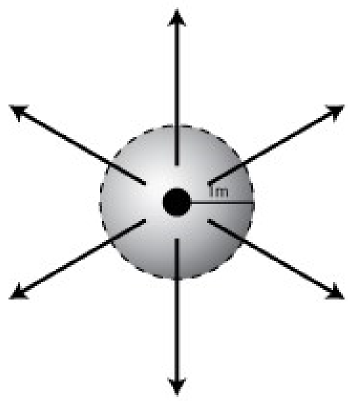
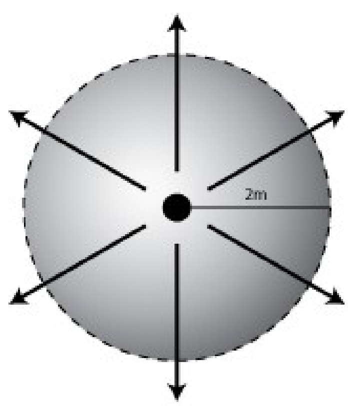


Fig. 3



In this article, we explained the inverse square law. Notice how the square came out. It is because the area of a sphere in a 3-dimensional space world is proportional to the radius squared. Similarly, if our space we live in were 4-dimensional, the gravitational force would be inversely proportional to the cube of the distance. Actually, string theorists assert that our space we live in is 10-dimensional,[1] but 7 of the 10 space dimensions are very small, and so we are left with only 3 of them if we only count the big ones. Therefore, even though the gravitational force may be inversely proportional to a power to the ninth degree of distance for small distances, it is inversely proportional to the square in our macroscopic world.

Final remark. we will revisit the inverse square law in our later article "Gauss's law." Gauss's law can be applied both to electric fields and to gravitational fields because they both follow inverse square law. This statement will be clear in the later article.

Summary

- The gravitational force and Coulomb force are inversely proportional to the distance squared.
- It is because the area of a sphere in a 3-dimensional space world is proportional to the radius squared.

[1] i.e. our spacetime we live in is 11-dimensional,