The law of cosines

If a and b are two sides of triangle, and the angle between them is $90°$ we know that the other side c is given by $\sqrt{a^2+b^2}$ from Pythagorean theorem. However, what if the angle between them is not $90°$ but another angle θ? This is the question to which we will find an answer in this article as promised in our earlier article "Pythagorean theorem." To this end, we will find the coordinates of each vertex using Cartesian coordinate system. Then, we will calculate the length of the other side using Pythagorean theorem. See Fig.1. If the vertex where the angle θ is located is at the origin, and the side with length b is situated on x-axis, the coordinates of the other two vertices are given by $(b, 0)$ and $(a\cos\theta, a\sin\theta)$. Therefore, from Pythagorean theorem, we have:

$$\begin{aligned} c^2 &= (b - a\cos\theta)^2 + (a\sin\theta)^2 \\ &= b^2 - 2ba\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta \\ &= a^2 + b^2 - 2ab\cos\theta \end{aligned} \tag{1}$$

This is the law of cosines.

Problem 1. Check that (1) reduces to Pythagorean theorem when $\theta = 90°$.

Summary

- The law of cosines gives the side of a triangle in terms of the lengths of the other two sides and the cosine of the angle between them.

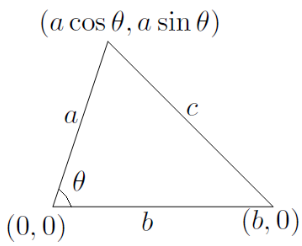

Figure 1: A triangle with side a, b, c