Length contraction paradox

In the last article, we have seen that the observation that other's clocks ticktock at slower rates than one's own led to a paradox, and how this apparent paradox was resolved, as it was not a bona fide paradox. In this article, we will concern the observation that other's rulers are shorter than yours (by Lorentz contraction) can lead to a paradox, and how this apparent paradox will be resolved, as it is not a bona fide paradox either. I first learned the paradox I will present here from Wolfgang Rindler's "Introduction to Special Relativity." Let's suppose an unrealistic situation in which 4 meter pole is Lorentz-contracted to 2 meter one as it moves very fast. And let's say that this pole enters a 2-meter garage, and you shut the door of the garage. See Fig. 1. You can shut the door, as the whole pole can go inside the garage.

On the other hand, the pole will see that the garage is Lorentz-contracted to 1 meter, as the garage is the one moving. See Fig. 2. Therefore, at first glance, it seems that the door cannot be shut. This is a paradox since whether the door can be shut or not should be observer-independent.

The resolution of this paradox lies in the fact that it takes time for the other end of pole to notice that the end of the pole reached the wall of the garage. The other end of the pole will keep moving until the shock wave from the end of the pole reaches it. The shock wave is way slower than the speed of light, and even if it is same as the speed of light, one can show that it is slow enough that the whole pole can go inside the garage, so that the door can be shut. Actually, the very fact that shock wave must first reach the other end of the pole for

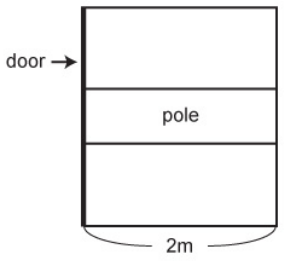

Figure 1: garage's perspective

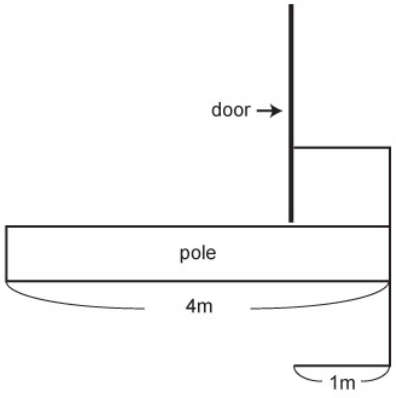

Figure 2: pole's perspective

it to stop moving is true in Fig. 1 as well. Therefore, the pole can be actually contracted to the size less than 2 meter.

In conclusion, the fact that there is no genuine rigid body resolved this paradox. By definition, a rigid body never changes its shape. If the pole were a rigid body, it cannot be contracted so short. In reality, it can be contracted as short as it can, until the shock wave arrives. In our daily lives, we don't usually notice the fact that there is no rigid body, because the shock wave is so fast that it is hard to notice that it doesn't take a zero time, but a finite time for the shock wave to reach the other end of the object. Therefore, a naive human may think that, if we have a long enough stick, say from here on the Earth to the Moon, and if you push the end of the stick here at the Earth, the other end of the stick in the Moon will move immediately. It doesn't. It takes time.

Summary

- There is no rigid body relativistically.