Light cone

Let's say that we have an event A and an event B. Let's consider their differences in spacetime coordinates. (Let's call it the "spacetime interval") We call a spacetime interval "spacelike" if it satisfies the following condition:

$$(\Delta x)^2 + (\Delta y^2) + (\Delta z)^2 - (c\Delta t)^2 > 0 \tag{1}$$

This terminology makes sense since the square of the space interval is bigger than the square of the time interval. Similarly, we call a spacetime interval "timelike" if it satisfies the following condition:

$$(\Delta x)^2 + (\Delta y^2) + (\Delta z)^2 - (c\Delta t)^2 < 0 \tag{2}$$

and "lightlike" for the following condition:

$$(\Delta x)^2 + (\Delta y^2) + (\Delta z)^2 - (c\Delta t)^2 = 0 \tag{3}$$

Let's think about some ramifications of these conditions. First, remember that $(\Delta x)^2 + (\Delta y^2) + (\Delta z)^2 - (c\Delta t)^2$ is invariant for any reference frame. If a spacetime interval is timelike, it is possible for a certain reference frame S' to measure the space interval to be zero (i.e. $\Delta x' = \Delta y' = \Delta z' = 0$) since the observer of reference frame S' could be at the spacetime position of A when A happens, and later move to the spacetime position of B when B happens. (If B happens earlier than A, then the observer could be at B and then move to A.) Certainly, in this case, the observer S' will think that both events A and B happened at the same place, namely where he is situated himself. On the other hand, measuring the time interval to be zero (i.e. $\Delta t' = 0$) would be impossible, as it would imply that the left-hand side of (2) is non-negative. (We always have $(\Delta x)^2 + (\Delta y^2) + (\Delta z)^2 \geq 0$.)

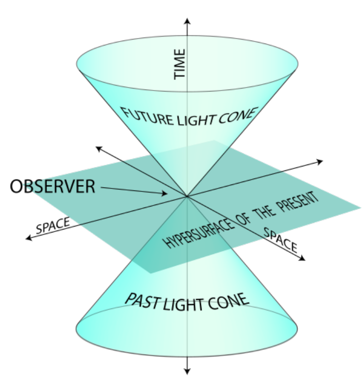

Figure 1: light cone

Similarly, if a spaceime interval is spacelike, it is possible to measure the time interval to be zero (i.e. $\Delta t' = 0$) while measuring the space interval to be zero (i.e. $\Delta x' = \Delta y' = \Delta z' = 0$) would be impossible, as in the latter case the left-hand side of (1) would be negative or zero. Moreover, the observer cannot move from A to B to observe that A and B happened in the same place since he cannot move faster than light.

Notice the following. If a spacetime interval of two events is timelike, no observer would be able to say that the two events happened at the same time. If, say, A happened earlier than B, this is something that all observers must agree to be the case.

On the other hand, if a spacetime interval of two events, say A and B, is spacelike, some observers would be able to say that the two events happened at the same time, which implies that all the observers cannot agree on whether A happened earlier than B or whether B happened earlier than A. In this case, the time sequence of two events is not absolute, but observer-dependent.

These ramifications suggest the picture of the light cone. See the figure. The x and y axes denote the space and the z axis denotes the time. The surface of the light cone is formed from the collection of points that satisfy (3). Let's say that an event A happened at the origin of the graph. If an event B happened inside the future light cone, the spacetime interval is timelike; then everybody can agree that event B happened after event A. Similarly, if an event B happened inside the past light cone, the spacetime interval is timelike; then everybody can agree that event B happened before event A. On the other hand, if event B happened outside of the two light cones, there could always be observers for whom event A happened first as well as those for whom event B happened first.

Problem 1. Let's say that event A is April 2018 inter-Korean summit in South Korea and event B is June 2018 North Korea-United States summit in Singapore. Is the spacetime interval between these two events spacelike or timelike?

Summary

- A spacetime interval between two events is spacelike if $(\Delta x)^2 + (\Delta y^2) + (\Delta z)^2 > (c\Delta t)^2$.
- Then, we can find a reference frame in which $\Delta t' = 0$.
- In such a case, some observers observe one of the events happened first, while others observe the other happened first.
- A spacetime interval between two events is timelike if $(\Delta x)^2 + (\Delta y^2) + (\Delta z)^2 < (c\Delta t)^2$.
- Then, we cannot find a reference frame in which $\Delta t' = 0$.
- In such a case, every observer agrees which of the two events happened first, and which of the two events happened later.

(The figure is from `http://en.wikipedia.org/wiki/File:World_line.svg`)