# How to turn a complicated multiplication into a simple addition

Let's say you want to multiply two complicated numbers, say, 8.54291 and 57.172. How would you do it? It's very easy. You can just use a calculator or a computer. However, how would you do it if you were born in the 17th century? Astronomers then, as now, often needed complicated computations, and it was not always easy to perform multiplications by hand. Luckily, Scottish mathematician and astronomer, John Napier invented a novel method for quick multiplication in 1614. What he actually used is called "logarithms," which you may be familiar with, if you learned it at high school. However, you may have found it boring and wondered why it would be useful. I also didn't know exactly about Napier's method of multiplication until recently. As it's interesting enough, I share it with you.

Suppose you want to multiply 32 by 8. This multiplication is easy to do by hand, but let's use a method similar to Napier's to illustrate his method. First of all, notice

$$32 = 2^5, \qquad 8 = 2^3 \tag{1}$$

Thus, we want to calculate

$$2^5 \times 2^3 = 2^{5+3} = 2^8 = 512 \tag{2}$$

See what we have done. We turned a multiplication "$32 \times 8$" into an addition "$5 + 3$".

Now, let's multiply our earlier numbers using Napier's method. First, note that

$$8.54291 \approx 10,000,000 \times 0.9999999^{139729933} \tag{3}$$

$$57.172 \approx 10,000,000 \times 0.9999999^{120720308} \tag{4}$$

As

$$139729933 + 120720308 = 260450241 \tag{5}$$

we have

$$8.54291 \times 57.172 \approx 10^7 \times 10^7 \times 0.9999999^{260450241} \tag{6}$$

$$\approx 10^{14} \times 4.8841522 \times 10^{-12} = 488.41522 \tag{7}$$

The real answer is about 488.41525. So, we see that Napier's method is very effective.

What Napier actually did was following. He considered the following formula.

$$N = 10,000,000 \times 0.9999999^L \tag{8}$$

i.e., (3) and (4). When $L = 0$, $N$ is 10,000,000, and when $L = 1$, $N = 9,999,999$ and so on. He repeatedly multiplied 0.9999999 to get the values of $N$ for different values of $L$. He made the result of this calculation into tables so that one can find a corresponding value of $L$ for $N$ between 5 and 10,000,000. Actually, he used some tricks in his calculation. For example, instead of multiplying 0.9999999 every time, he multiplied numbers such as

$$0.9999999^{100}, \qquad (0.9999999^{100})^{20}, \qquad ((0.9999999^{100})^{20})^{20} \tag{9}$$

It took him 20 years of calculation. Of course, if he didn't use these tricks, it would have taken longer.

In our later article "Logarithms," we will talk more about logarithms. There, I will also explain what kind of wrong guess I had about John Napier's invention and method upon learning that he used logarithms for calculation. We will also let you figure out how and why a slide-ruler, a ruler used for multiplication, works.

**Problem 1.** We have seen how we can multiply numbers by using Napier's method. Then, how can you divide by Napier's method? Explain.