Lorentz-Fitzgerald contraction: Why is a moving object shortened?

According to Einstein's theory of special relativity, an object moving very fast gets shortened along the direction it is moving - a phenomenon called "Lorentz-Fitzgerald contraction." In this article, I will show why this must be the case, by using the concept of time dilation that I introduced in my articles "Time dilation in Einstein's theory of special relativity" and "Electromagnetic forces and time dilation in physics." In this present article, I will closely follow the discussion presented in "Concepts of Modern Physics" by Beiser, since I found his argument easy to follow.

Let's say that you observe a particle called a "muon" coming straight down from the sky with a speed v. Let's also say that the lifetime of a muon at rest is t, and that when it arrives at the ground where you are standing, it decays. From your perspective on the ground, its lifetime will be not t, but longer, since a moving object's time flows more slowly than the one at rest. According to the special relativity, the lifetime of muon you will observe is given by the following formula:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

where c is the speed of light. Notice that this denominator is smaller than 1, which is consistent with the lifetime of muon you observe being bigger than the lifetime t of the muon as observed by the muon itself. (From the perspective of the muon, the muon is not moving.)

Given the speed and lifetime observed by you, you will see that the muon travels by the distance vt'. Writing this explicitly, the distance that you observe the muon to travel is given by:

$$vt' = \frac{vt}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

However, according to the muon, the muon is not the one who traveled; you are. Would the muon also think that you travelled by the distance vt'? No. Since the muon will observe that you have travelled with speed v for a time

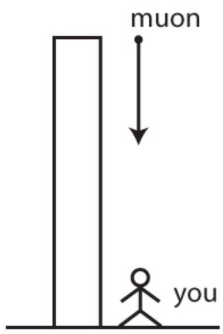

Figure 1: your perspective

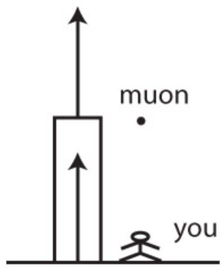

Figure 2: muon's perspective

t rather than t', the muon will observe that you travelled by vt. Of course, these distances are related by the following formula:

$$vt = vt'\sqrt{1 - \frac{v^2}{c^2}} \tag{3}$$

Think of these distances as the spaces observed between you and the muons initial point. The muon will see that the space between its initial point and you is moving and is shortened by the factor given by above formula. On the other hand, you will see that the space between the initial point of muon and you is not moving and is not shortened.

Let me clarify my argument. See Fig. 1 and Fig. 2. Suppose that you have a ruler that fits between yourself and the initial point of the muon. Then, you will not see this ruler moving, as one of the two tips of the ruler will be fixed at your position. However, from the point view of the muon, the ruler will be moving, since one of the two tips of the ruler will be at your position and muon will see you moving in its direction. The ruler will also look shorter to the muon than it looks to you. Therefore, a moving object is shortened. If the length of the ruler at rest is given by L_0, the muon will see that the length of the ruler is given by

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} \tag{4}$$

Notice that unless the object moves very very fast, v/c is small and the factor by which it gets shortened is very close to 1. For this reason, we never notice Lorentz-Fitzgerald contraction in our daily lives, any more than we notice time dilation in our daily lives.

Final comment. The idea for Lorentz-Fitzgerald contraction (4) was first conceived by Irish physicist Fitzgerald in 1889. He argued that the reason why Michelson-Morley experiment failed to detect Earth's movement with respect to ether is because length is contracted in the direction of motion, compensating the speed of light difference due to ether wind. However, he also noted that it would be impossible to detect this contraction, because the ruler to measure this contraction would be contracted in the same proportion. In 1892, Dutch physicist Lorentz also independently came up with the idea of contraction. However, their formula was not (4), but the approximation of (4), namely, $L = L_0\left(1 - v^2/2c^2\right)$. It was Einstein who found (4), and correctly interpreted it: Lorentz contraction of an object occurs not because of its relative motion to ether, because there is no ether.

Summary

- A moving object is shortened. If the length of the object in its rest frame is L_0, the length observed by an observer moving with speed v relative to the observer is given by

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$