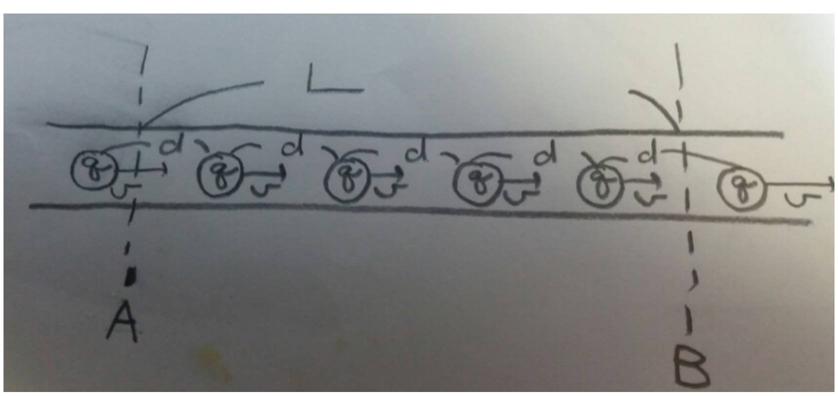
# Magnetic force on a current-carrying wire

In the last article, we learned that the magnetic force on moving charge is given by $q\vec{v}\times\vec{B}$. However, this formula is not convenient to calculate the magnetic force on a current-carrying wire, as the velocity of moving charge inside the wire is not directly measurable. Instead, the electric current, defined by electric charge passing through a cross section of wire per unit time is directly measurable using device such as amperemeter. So, in this article, we will find the magnetic force on a current-carrying wire in terms of $i$, the electric current and $L$, the length of the segment on which the concerned magnetic force is exerted. We will first consider the case that $v$ and $B$ are perpendicular, and later generalize our results. Now, see the figure. We simplified the situation and assumed that every moving charge has the same charge $q$, the same speed $v$ and each of them is separated by the same distance $d$. There are other charges than the moving charges to make the wire neutral (i.e. no net charge), but we didn't draw them for simplicity. Given this, we want to calculate the magnetic force on the segment of the electric wire between the cross sections $A$ and $B$. The concerned length is $L$ as denoted in the figure. As there are $L/d$ moving charges in this segment, the total electric charge is $qL/d$. Therefore, the magnetic force is given by

$$F = (qL/d)vB \tag{1}$$

Now, let's express the electric current using these variables. The charge $q$ passes the cross section $A$ (or $B$, it doesn't matter which one) every $d/v$ seconds. Thus, on average, the amount of charge passing through the cross section $A$ per unit time is given by $i = q/(d/v)$.



Thus, the above expression can be re-expressed as

$$F = iLB \tag{2}$$

For a general case (i.e. the one corresponding to $\vec{F} = q\vec{v} \times \vec{B}$), we have

$$\vec{F} = i\vec{L} \times \vec{B} \tag{3}$$

where $\vec{L}$ is the vector along the wire pointing in the direction of current. It has the same direction as $q\vec{v}$

This is our final result. Notice that (3) doesn't depend on $q$, $d$ or $v$. Of course, in reality, the moving charges do not have the same speed, nor are they separated by the same distance, but you can regard $v$ as the average speed, and $d$ as the average separation distance. In any case, the above expression still doesn't depend on $q$, $d$ or $v$. Of course it doesn't depend on the sign of $q$ either. Actually, the electric current is carried by electrons which have negative charges. Therefore, the direction of the electrons moving and the direction of electric current are opposite to each other. Still again, (3) is satisfied.

## Summary

- From $\vec{F} = q\vec{v} \times \vec{B}$, we can derive

  $$\vec{F} = i\vec{L} \times \vec{B}$$

  where $\vec{F}$ is the magnetic force exerted on the segment length $L$ of the current carrying wire, $i$ the current, and $\vec{B}$ the magnetic field. $\vec{L}$ is the vector along the wire pointing in the direction of current.