English as a foreign language versus English as a mother tongue: an analogy for the language of mathematics and the language of physics

English was my first foreign language. It was very difficult. It took me a long time to master it. The grammar has many structures that are not present in Korean, my mother tongue, and I had to memorize many things. For example, English speakers use "to" or "ing" to connect two verbs. These can be used interchangeably in some cases, but not in all. For example:

> Jason forgot calling Amy.
> Jason forgot to call Amy.

The first sentence implies that Jason forgot the fact he had called Amy, while the second sentence implies that Jason forgot the fact that he was supposed to call Amy, which in turn implies that he didn't call her.

Learning English took me a long time, as I had to memorize things like this. Native English speakers know all these things even though they never learned them intentionally; they picked up the usages of "to" and "ing" outside of a classroom. Yet, they learned English much more quickly and speak English much more properly than non-native speakers like me.

A similar situation exists regarding the languages of mathematics and physics. When I was young, I asked a string theorist which math courses he would recommend me to take. He replied that in retrospect only the mathematics courses designed for physics students were helpful for him, as a physics student can learn mathematics much more quickly than a mathematics student.

This is true. Mathematicians place significant emphasis on rigorous foundations by carefully proving every assertion in their argument, which takes a long time, whereas physicists place a lot of emphasis on physical clarity and intuition, even though these may come at the cost of some mathematical rigor. For example, unlike mathematicians, physicists do not bother to prove obvious theorems.

An example of obvious theorems would be Jordan curve theorem. Jordan curve theorem says that a non-self-intersecting continuous loop (called "Jordan curve") in the plane divides the plane into an interior region and an exterior region.

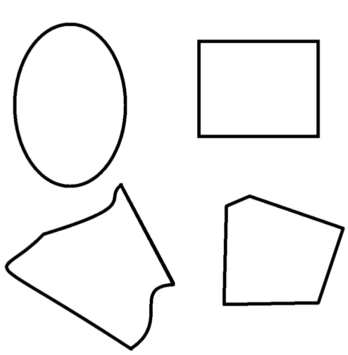

Figure 1: examples of Jordan curves

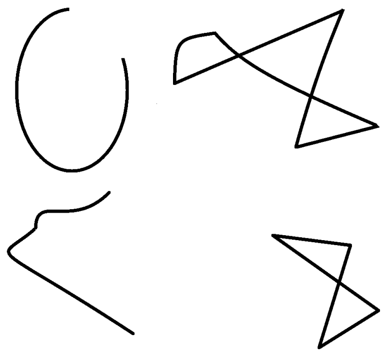

Figure 2: examples of non-Jordan curves

See Fig. 1 for examples of Jordan curves, and see Fig. 2 for examples of non-Jordan curves. It is intuitively obvious that a Jordan curve divides the plane into an interior region and an exterior region, while this is not necessarily true for non-Jordan curves.

While the assertion of Jordan curve theorem is obvious, it is not easy to prove it. I cannot prove it myself, let alone understand the proof. Mathematicians care about such proofs, because there should be no single loophole in their logic. Otherwise, the system they constructed, called "mathematics" is not on a firm foundation. On the other hand, physicists are more carefree. To begin with, physical laws are not certain, because they are based on experiments and observations that necessarily have measurement errors. Einstein said "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."

Furthermore, the languages of mathematics and physics are very different. The language of mathematics itself reflects the fact that mathematicians like rigor. The Russian mathematician V. I. Arnold wrote:

> It is almost impossible for me to read contemporary mathematicians who, instead of saying "Petya washed his hands," write simply: "There is a $t_1 < 0$ such that the image of t_1 under the natural mapping $t1| \rightarrow \mathrm{Petya}(t_1)$ belongs to the set of dirty hands, and a t_2, $t_1 < t_2 \leq 0$, such that the image of t_2 under the above-mentioned mapping belongs to the complement of the set defined in the preceding sentence.

What the above statement in mathematical language essentially means is that Petya's hands were dirty before the time was t_1, but they became not-dirty after the time was t_2 where $t_2 > t_1$ (i.e t_2 was later than t_1). Also, we have $t2 \leq 0$, since 0 is the present time and Petya washed his hands in the past.

This is perhaps the reason why I like physics more than math; not only am I not very good at reading mathematical sentences like this, but I simply don't want to bother myself with proving things that are obvious. I feel no need to know the proof for something such as the de Rham theorem that seems "correct" from intuitive reasoning.

In other words, I feel like I am one of the physicists described in the introduction of the book Mirror Symmetry: "[Physicists] feel that attempts to build on a more rigorous foundation, while noble, will distract them from their goal of understanding nature." I feel that all of the complicated-looking proofs hamper me from seeing whether they are just proofs for something obvious or, instead, non-trivial and insightful proofs attacking the crux of problems.

Mathematician and physicist Arthur Jaffe and mathematician Frank Quinn begin their article "Theoretical Mathematics: toward a cultural synthesis of mathematics and theoretical physics" by noting that the use of rigorous proofs in mathematics makes itself "slow and difficult" even though it brings "reliability unmatched by any other science." They then define their use of the terms "theoretical mathematics" and "rigorous mathematics" as follows:

> Typically, information about mathematical structures is achieved in two stages. First, intuitive insights are developed, conjectures are made, and speculative outlines of justifications are suggested. Then the conjectures and speculations are corrected; they are made reliable by proving them. We use the term theoretical mathematics for the speculative and intuitive work; we refer to the proof-oriented phase as rigorous mathematics.

They also note:

> The initial stages of mathematical discoverynamely, the intuitive and conjectural work, like theoretical work in the sciences involves speculations on the nature of reality beyond established knowledge.

Then they compare proofs in mathematics with experiments in other sciences; one needs to check whether a theory is true through a proof. Moreover, a proof, like an experiment, can provide new insight and new possibilities as by-products.

After giving some examples of "theoretical" mathematics, they note:

> Another type of mathematical work is intermediate between traditional and theoretical. It proceeds in the way, "If A is true, then X, Y, and Z follow", or "If A, then it is reasonable to conjecture R, S, and T."

The article concludes as follows:

> At times speculations have energized development in mathematics; at other times they have inhibited it. This is because theory and proof are not just “different” in a neutral way. In particular, the failure to distinguish carefully between the two can cause damage both to the community of mathematics and to the mathematics literature. One might say that it is mathematically unethical not to maintain the distinctions between casual reasoning and proof. However, we have described practices and guidelines which, if carefully implemented, should give a positive context for speculation in mathematics.

Jaffe and Quinn seem to suggest that doing mathematics the way a physicist would can be very useful for mathematicians. I agree, possibly because I myself am a physicist; it would take me too long to learn math in the same way some non-native speakers learn English, emphasizing too much on grammar. I would like to be a native speaker of mathematics who did not worry about grammar, even at the risk of making mistakes sometimes that I would not be able to catch. (Native speakers of English sometimes make mistakes, and never having learned grammar in detail, they can't tell themselves whether they are mistakes in certain cases!)

I think the following quote summarizes the content of this article well. Citing a “cultural gap,” MIT physicist Max Tegmark noted: “Some mathematicians look down their nose at physicists for being sloppy−for doing calculations that lack rigor.” He also noted: “You guys take forever to derive things that we can get in minutes. And if you had our intuition, you'd see it's all unnecessary.”