Why Math Education Matters

"Suppose a belt tightly stretched about the equator of the earth just fits. How long a piece should be inserted so that the belt could encircle the earth at a distance of 1m away at all points?"

The answer to this frequently posed math problem is 6.28 m. It may seem strange that the answer is so small, compared with Earth's circumference, but the math behind the solution is straightforward. Let the radius of Earth be r meters. Then the belt that fits the equator of the earth will be $2\pi r$ meters. To encircle the earth at a consistent distance of 1m away, the belt will have to be $2\pi (r+1)$ m. The difference between $2\pi (r+1)$ m and $2\pi r$ m is 2π m or 6.28 m.

When I decided not to eat non-cage-free chickens, some people told me that my decision would not have a non-negligible effect upon the supply of non-cage-free chickens. I also heard someone saying that she invested in the stock market because the stock prices often rose several percent a *week* while the interest rate was only several percent a *year*. She ignored the fact that stock prices often drop several percent a week, which makes it extremely difficult for the stock prices she bought to rise several percent a week on *average* for a long period of time. I have a feeling that these people are among those who, in the above problem, would be likely to imagine that a piece much longer than 6.28 m would need to be inserted into the belt.

I imagine that these people, by the same token, are also likely to think erroneously that power plants do not need to increase their production to counteract the fact that they themselves waste energy since the amount of energy that they consume is negligible compared to the amount of surplus energy the government prepares to prevent blackouts. I also read that a mayor of a small village in Spain squandered their annual budget to buy the lottery tickets in the hope of paying 1 million euro debt. Perhaps, such a mistake is not uncommon. A Korean thief made the same mistake with the money he had stolen. I am sure that the mayor and the thief would not be able to solve the above math problem.

On Internet, I read a question asking why physicists still operate the Large Hadron Collider (LHC) at CERN, even though there does exist a non-zero possibility that harmful black holes will be formed, no matter how small such a probability may be. I had to answer this question by asking why the questioner takes cars even though it is much more dangerous than the LHC. I am sure that the questioner would not be able to solve the above math problem.

On Internet, I read a comment that a wife had added only 1/3 cup of sugar instead of 1/4 cup as in the recipe because her husband didn't like things too sweet, and it turned out to be perfect for

them. Apparently, she didn't know that the number 1/3 is bigger than the number 1/4.

It seems that this wife was not the only one who didn't know this. In the 1980s, A&W Restaurants promoted their third-pound burger aggressively to compete against McDonald's Quarter Pounder. The third-pound burger outperformed in the blind taste tests and was set at the same price as their competitor's. However, the marketing was not successful. To find out why their marketing failed, they asked a market research firm. By conducting a survey, the market research firm found out that the customers were concerned about the price of the third-pound burger. As the majority of survey participants mistakenly thought that a third-pound burger weighs *less* than a quarter-pound burger, they were not willing to pay the same price for the third-pound burger as the quarter-pound burger.¹

Math education matters, because the general public, as well as people in influential positions such as mayors or CEOs, should not make mistakes in cases such as these. I am very sorry that the action of the mayor of the Spanish village resulted in more debt for his village instead of less debt. It is also important that the tax-payers who fund scientific research such as the operation and the construction of LHC understand that their money is used in harmless research. I also feel very angry when lottery number prediction companies rip off poor people by lies such as "Secrets to the lottery number discovered!" or "There are reasons why you have never won even the smallest prize amount in the lottery" or "Don't throw away your lottery tickets even though they didn't win."

Many people, including me, harshly criticize the South Korean math education; mathematics is taught in boring ways, students are expected to memorize unnecessarily many mathematical formulas, and too high math level is expected for the least talented students. According to a survey, about 20% of elementary school students (grade 1-6), about 40% of middle school students (grade 7-9), and about 60% of high school students (grade 10-12) have given up mathematics. This shows the seriousness of the South Korean math education problem.

On the other hand, even these least mathematically talented students in Korean standards are able to tell right away that 1/3 is bigger than 1/4, which every cook, every househusband, every housewife, and every burger lover must know. As another example, some people in the West claim that vaccines for COVID-19 are useless because those who are vaccinated can also get

¹ The Truth About A&W's Third-Pound Burger and the Major Math Mix-Up

https://awrestaurants.com/blog/aw-third-pound-burger-fractions

COVID-19. Yet, I have never met a Korean, who says that vaccines are useless. Most, if not all, Koreans were so eager to get vaccinated as soon as they got available.

A vast majority of people have a misconception about "being good at math." They think that "being good at math" means "being able to multiply two 2-digit numbers in your head." But that means being good at arithmetic, not at math. It doesn't require multiplication of two 2-digit numbers to figure out that the supply of non-cage-free chickens will diminish if you don't eat them, to understand that there are no reasons why you have never won even the smallest prize amount in the lottery, and to spot what is wrong with the argument that vaccines are useless.

It seems that juries, too, need to have the mathematical mindset to spot out the error in the argument of attorneys. O. J. Simpson, the former American football star was charged for the murder of his former wife. Against the accusation of the prosecutors that Simpson often beat his former wife, his attorney argued that the accusation was irrelevant considering that fewer than 1 out of 2,500 husbands who beat their wives end up murdering them. A convincing argument, isn't it? Isn't there only 1 out of 2,500 (i.e., only 40 out of 100,000) chance that Simpson killed his former wife? No, contended I. J. Good, the British statistician.² He pointed out that the correct question to ask is: What is the probability that a husband murdered his wife, given that he had physically abused her, and his wife had been already murdered? In the US, 5 out of 100,000 women are murdered, whether by their husbands or by someone else. Now, think of a group of 100,000 American women, who suffer under the physical violence of their husbands. Out of these 100,000 American wives, 40 are killed by their husbands, as we just mentioned. How many of these 100,000 abused American wives are killed by someone other than their husbands? Most likely, at most 5. Therefore, at most, 45(=40+5) out of 100,000 American wives abused by their husbands are killed, and among those, about 40 wives are killed by their husbands. The probability that a woman who was physically abused by her husband, and murdered, was actually killed by her husband is therefore around 40/45, which translates to about 90%.

I read this explanation by the British statistician from the Korean translation of "数学の言葉で世界 を見たら 父から娘に贈る数学" (If we look at the world through the language of mathematics, mathematics presented by a father to a daughter), the book written by the Japanese string theorist Hirosi Ooguri for a wide audience. While this calculation of the probability cannot and

² I.J. Good, "When batterer turns murderer," Nature, Vol. 375 (1995), p. 541. I.J. Good, "When batterer becomes murderer," Nature, Vol. 381 (1996), p. 481.

should not find Simpson guilty with 100 % certainty, it shows that the argument Simpson's attorney made must be rejected with 100 % certainty.

We teach mathematics to our children, not because they will be mathematicians, but because we need to give them mathematical insight so that they can look at the world through the language of mathematics. When people have the knowledge and skills necessary to understand critical issues from a mathematical point of view, they will be able to make better-informed decisions about their social, economic, medical, and legal behaviors and practices.