Maxwell's equations in matter

In earlier articles, we have learned Maxwell's equation. However, it turns out that there is a more convenient version of Maxwell's equations when concerning to matter. For example, see Fig.1. A ball with a positive charge Q_{free} is immersed in a certain medium, such as air or water. (It will be apparent later why we call it "free.") However, as matter is composed of electrons and nuclei, the electric force due to the charge Q_{free} will attract the electrons and repel the nuclei. Therefore, we will see the negative charge $-Q'$ cluster around the ball. Therefore, the electric field at the point distance r far from the center of the ball (r bigger than the radius of the ball) will be

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q_{\text{free}} - Q'}{r^2} \tag{1}$$

or, equivalently

$$\epsilon_0 \int \vec{E} \cdot dA = Q_{\text{free}} - Q' \tag{2}$$

Also, it is actually easy to imagine that $-Q'$ is proportional to Q. The bigger the positive charge the more it attracts the negative charge. So, we can express this relation, say, as $Q' = aQ_{\text{free}}$ for certain a that depends on the medium. Then, we can re-express the above equation as

$$\epsilon \int \vec{E} \cdot dA = Q_{\text{free}} \tag{3}$$

where we used the following definition.

$$\epsilon = \frac{\epsilon_0}{1-a} \tag{4}$$

As a depends on the medium ϵ must depend on the medium as well. Therefore, Maxwell's equation changes in a medium. Actually, it doesn't change, but if we consider "free charge" such as Q_{free}, it is more convenient to express Maxwell's equations in a changed form. So

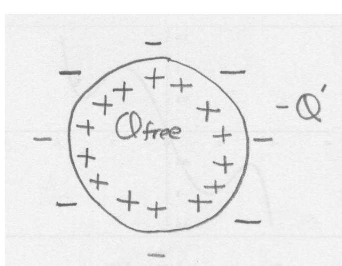

Figure 1: a charged ball in a medium

far, we talked only about electric field. A similar analysis can be done for magnetic field, and the result is that μ_0 is replaced by μ, and electric current is replaced by a free one. Then, recalling how the speed of light was calculated in an earlier article, it goes without saying that the speed of light in a certain medium is given by

$$v = \frac{1}{\sqrt{\epsilon\mu}} \tag{5}$$

where ϵ and μ are the ones for that medium.

Let me conclude this article with a historical remark. Maxwell published Maxwell's equations in 1861 and obtained the speed of light in 1862. In 1875, Lorentz got a Ph.D for deriving Snell's law by using Maxwell's equations in a matter, such as the one we presented in this article. We will not show you the derivation.

Summary

- In the presence of medium, the electric field is weakened, because the opposite charge clusters around the free charge.

- Even in the presence of medium, the Maxwell equations still hold. However, if we consider only free charge, Maxwell equations must be modified in such a way that charge is replaced by free charge, and ϵ_0 by ϵ and μ_0 by μ. Thus, the speed of light in the medium is given by $v = \frac{1}{\sqrt{\epsilon\mu}}$.