M-theory and dualities

When string theorists first found that there are five consistent string theories, it looked troublesome. It didn't make sense that there would be more than one consistent string theory. In other words, at first glance, if one of them were correct, the other four must be ruled out.

However, string theorists soon realized that all of these five string theories can be correct. They realized that each one is a different limit of the more fundamental "M-theory."

To describe what I mean by "different limits," let me give you an example by saying that theory A and theory B are two different limits of theory AB which is more fundamental (of course, I am just making these up for heuristic purposes). Let's assume the following:

- Theory A: $y = x^2 + 1$
- Theory B: $y = 1 + \frac{1}{x^2}$

As we see above, the quantity y in theory A and theory B is different. However, these could be two different limits of the following theory AB:

- Theory AB: $y = x^2 + 1 + \frac{1}{x^2}$

If x is much larger than 1, $x^2 + 1 + \frac{1}{x^2}$, the quantity y in theory AB can be approximated by $x^2 + 1$, the quantity y in theory A. If $x = 100$, then $x^2 + 1$, the value y in theory A, is equal to 10001 while $x^2+1+\frac{1}{x^2}$, the value y in theory AB, is 10001.0001. The two are really close. As x gets bigger, the difference between $x^2 + 1 + \frac{1}{x^2}$ and $x^2 + 1$ becomes ever smaller. Therefore we can say that theory A is a large-x limit of theory AB.

Similarly, one can show that theory B is a small-x limit of theory AB. For example $x = 0.001$ yields y=1000001 in theory B and y=1000001.000001 in theory AB. Again, the difference between y in theory B and y in theory AB gets smaller and smaller as x gets smaller and smaller. Summarizing we get:

- Theory A: $y = x^2 + 1$ (valid for $x \gg 1$)
- Theory B: $y = 1 + \frac{1}{x^2}$ (valid for $x \ll 1$)
- Theory AB: $y = x^2 + 1 + \frac{1}{x^2}$ (valid for all x)

Now we can understand what is meant by "different limits."

Evidences for M-theory were obtained as early as the mid 90s, but its full equations have not yet been discovered. It's as if we know $y = x^2 + 1$ and $y = 1 + \frac{1}{x^2}$ but don't yet know $y = x^2 + 1 + \frac{1}{x^2}$.

Making the relationship between these five different string theories and M-theory more interesting are dualities between string theories. Dualities relate different string theories in an interesting way. For example, theory A and theory B are dual to each other, as when we substitute $\frac{1}{x}$ for x in theory A, we get theory B and vice versa. The value of y in theory A when x is $\frac{1}{x'}$ is equal to the value of y in theory B when x is x': $y = (1/x')^2 + 1 = 1 + \frac{1}{x'^2}$. One important property of duality is that you recover the original theory if you take the dualities twice. (We briefly showed such an example in our earlier article "Linear inequalities.") In our case, the duality is given by $x \to \frac{1}{x}$. If you take the duality once again, we get $\left(\frac{1}{x}\right) \to \frac{1}{\left(\frac{1}{x}\right)} = x$, so we go back to the original theory.

The evidences for M-theory were unveiled through the discovery of the dualities in string theory in the mid 90s. This discovery is called the second superstring revolution (the discovery of the five superstring theories in the mid 80s is the first superstring revolution).

In fact, we can state what the actual dualities among the five string theories are. To this end, we first need to introduce string coupling constant g_s, which plays a pivotal role in string theory. String coupling constant dictates how strong strings interact from one another.

"S-duality" is a kind of the duality that we have considered in our example. In our example, we had $x \leftrightarrow \frac{1}{x}$. The S-duality in string theory is given by $g_s \leftrightarrow \frac{1}{g_s}$. More generally, S-duality is given by $g \leftrightarrow \frac{1}{g}$ where g is a coupling constant. Seiberg duality we talked about in "Linear inequalities" is an example of S-duality, even though we didn't show that it was an S-duality. In our later articles, we will see that the duality between the electric field and the magnetic field is also S-duality.

"T-duality" is a duality about the size of the circle in the extra dimensions. As we have not taught you what the extra dimensions are, we will postpone our discussion of T-duality to our later article "Manifold." In any case, T-duality is given by $R \leftrightarrow \frac{1}{R}$ where R is the size of the circle in the extra dimensions.[1]

Now, let's state the actual dualities among the five string theories. The five string theories are: "type I," "type IIA," "type IIB," "SO(32) heterotic," and "$E_8 \times E_8$ heterotic."

There is an S-duality between type I and SO(32) heterotic. Also, type IIB has an S-duality with itself. (It's like "theory AB" in our example; upon $x \to \frac{1}{x}$ theory AB goes to itself.) There is a T-duality between type IIA and type IIB, and another T-duality between type I and SO(32) heterotic.

Summary

- Five string theories are different limits of M-theory.
- A duality is given by $x \leftrightarrow \frac{1}{x}$.

[1]Strictly speaking, it should be $R \leftrightarrow \frac{l_s^2}{R}$ where l_s is called "string length scale."

- The five string theories are related to one another by dualities.