Natural units

Roughly speaking, natural units are convention used in physics that can express complicated formulas a bit simpler by omitting certain important constants by setting them equal to 1. These constants are usually c, $\hbar$, G and k. (Here, k is the Boltzmann constant, and $\hbar$ the reduced planck constant. Don't worry if you don't know them. We will encounter them in later articles.) For example, instead of writing the following equations in Lorentz transformation,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{1}$$

$$t' = \gamma(t - \frac{vx}{c^2}) \tag{2}$$

one simply writes:

$$\gamma = \frac{1}{\sqrt{1 - v^2}} \tag{3}$$

$$t' = \gamma(t - vx) \tag{4}$$

Using natural units is especially useful when you calculate things manipulating variables by hand. You can forget whether c is multiplied to this term or that term is divided by c^2. Then, when you finally obtain your formula expressed in natural units, such as the ones in (3) and (4), you can convert them to the ordinary ones such as (1) and (2) by comparing the dimensions of each term.

Let's see how it works in our cases. In (3), we know that γ and the numerator of the right-hand side are dimensionless. Therefore, the denominator, as well as the term inside the square root of the denominator should be dimensionless. However, while 1 is dimensionless v^2 is not dimensionless. So, we have to multiply or divide some powers of c to make it dimensionless. As v^2 has dimensions of speed squared, we have to divide by the speed squared to make it dimensionless. As c has the speed as its dimension, we have to divide by c^2. This results in (1).

Similarly for the second case. As the left hand side of (4) has the dimension of time, the right hand side must have the dimension of time as well. However, while the first term in the parenthesis of the right hand side correctly has time as its dimension, vx has the dimension of $\left[L^2T^{-1}\right]$ where L denotes the length and T denotes time, as it is distance multiplied by speed. So, if we divide vx by c^2, we will have T as its dimension, matching with the left-hand side.

Problem 1. According to special relativity, the energy of a particle E and its momentum p satisfy the following relation. (We will derive this formula in a later article.)

$$E^2 = m^2 + p^2 \tag{5}$$

where m is the mass of the particle (strictly speaking, the rest mass). Here, we used the natural units, i.e., $c = 1$. Restore the c factors in the above formula.

Summary

- Natural units are convention that expresses complicated formulas a bit simpler by omitting certain important constants such as c, $\hbar$, G, and k by setting them equal to 1.