The neutral kaon system

As mentioned at the last article, there are two kinds of neutral kaons: K^0 and $\bar{K}^0$, which form particle-antiparticle pair. As mentioned there, they are indeed distinct. To add some remark, while $\bar{K}^0$ can collide with matter to produce Λ^0, K^0 never can. For example,

$$\bar{K}^0 + p \to \Lambda^0 + \pi^+ \tag{1}$$

is possible, but $K^0 + p \to \Lambda^0 + \pi^+$ is never possible. It's because p (proton) and π^+ have strangeness 0 and Λ^0 has strangeness -1 and the above process is due to strong interaction which preserves strangeness.

However, when left alone, K^0 decays to two pions by weak interaction as follows

$$K^0 \to \pi^+ + \pi^- \quad \text{or,} \quad K^0 \to 2\pi^0 \tag{2}$$

Notice that the electric charge is conserved. But, the strangeness is not conserved as pions have strangeness 0. It is because weak interaction doesn't conserve strangeness. Similarly, $\bar{K}^0$ decays to two pions as follows

$$\bar{K}^0 \to \pi^+ + \pi^- \quad \text{or,} \quad \bar{K}^0 \to 2\pi^0 \tag{3}$$

Again, the electric charge is conserved, but the strangeness is not conserved.

Now, we can explain physicists Murray Gell-Mann and Abraham Pais's work. (Gell-Mann later won Nobel Prize for his quark model and Pais later became a very renowned physics historian.) K^0 can turn into 2 pions which then turn into $\bar{K}^0$. Similarly, $\bar{K}^0$ can turn into 2 pions which then turn into K^0. We can write it as

$$K^0 \rightleftharpoons 2\pi \rightleftharpoons \bar{K}^0 \tag{4}$$

Thus, there is a non-zero amplitude for K^0 to change into $\bar{K}^0$ as follows:

$$A = \langle K^0 | H | \bar{K}^0 \rangle \tag{5}$$

Similarly, there is an amplitude for $\bar{K}^0$ to change into K^0. By the symmetry between K^0 and $\bar{K}^0$, the amplitude must be equal to (5) as follows:

$$A = \langle \bar{K}^0 | H | K^0 \rangle \tag{6}$$

At this point, you may wonder that (6) should be the complex conjugate of (5) instead of being the same. However, when particles decay, the total number of particles is not conserved, which implies the time evolution operator $e^{-iHt/\hbar}$ is not unitary. Thus, H is not Hermitian.

Given this, let's write the Schrödinger equation for K^0 and $\bar{K}^0$. If we write

$$E = \langle K_0|H|K_0\rangle \tag{7}$$

we have

$$\begin{aligned} i\hbar\frac{d|K^0\rangle}{dt} &= H|K^0\rangle \\ &= (|K^0\rangle\langle K^0| + |\bar{K}^0\rangle\langle\bar{K}^0|)H|K^0\rangle && (8) \\ &= E|K^0\rangle + A|\bar{K}^0\rangle && (9) \end{aligned}$$

where in (8) we used the completeness relation to express the identity matrix. Of course, the neutral kaons decay into other products, so there should be other product states in the completeness relation, but we have taken into this account by making H non-Hermitian appropriately.

Similarly, using

$$E = \langle\bar{K}_0|H|\bar{K}_0\rangle \tag{10}$$

we have

$$i\hbar\frac{d|\bar{K}^0\rangle}{dt} = E|\bar{K}^0\rangle + A|K^0\rangle \tag{11}$$

Finally, subtracting (11) from (9), we get

$$i\hbar\frac{d(|K^0\rangle - |\bar{K}^0\rangle)}{dt} = (E - A)(|K^0\rangle - |\bar{K}^0\rangle) \tag{12}$$

adding (9) and (11), we get

$$i\hbar\frac{d(|K^0\rangle + |\bar{K}^0\rangle)}{dt} = (E + A)(|K^0\rangle + |\bar{K}^0\rangle) \tag{13}$$

Thus, if we define

$$|K_1\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}, \qquad |K_2\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}} \tag{14}$$

We have

$$|K_1(t)\rangle = e^{-i(E-A)t/\hbar}|K_1(0)\rangle \tag{15}$$

while

$$|K_2(t)\rangle = e^{-i(E+A)t/\hbar}|K_2(0)\rangle \tag{16}$$

Here, we see that K_1 and K_2 are eigenstates with definite energy. It means that they are the eigenstates of definite mass and definite lifetime;

The real part of $(E - A)$ is related to the mass of K_1 and the imaginary part of $(E - A)$ is related to the lifetime of K_1. And, similarly, the ones of $(E + A)$ to the ones of K_2.

Problem 1. Obtain the lifetime of K_1 in terms of $\mathrm{Im}(E - A)$.

Problem 2. Obtain the mass difference between K_1 and K_2 in terms of $\mathrm{Re}A$.

The actual mass difference between K_1 and K_2 are very tiny. It is about 3.5×10^{-12}MeV, while their mass is about 498 MeV.

Also, as they are eigenstates of definite lifetimes, their decay mode is different. K_1 decays into two pions, while K_2 decays into three pions.

Notice further following:

$$|K_0\rangle = \frac{|K_1\rangle + |K_2\rangle}{\sqrt{2}}, \qquad |\bar{K}_0\rangle = \frac{-|K_1\rangle + |K_2\rangle}{\sqrt{2}} \tag{17}$$

This means that if we produce pure K_0, only half of them decay into two pions, and the other half into three pions. You may wonder then which are "true" particles. K_0 and $\bar{K}_0$? or K_1 and K_2? Textbooks that I consulted say that both points of view are equally valid, while Gell-Mann and Pais say that K_1 and K_2 are, because they have definite life-times.

Even though we presented the neutral kaon system in a pedagogical manner, this was not the way Gell-Mann and Pais's original work was presented. Let me mention how they presented. If we denote the charge conjugation operator by C, we have

$$C|K_0\rangle = |\bar{K}_0\rangle, \qquad C|\bar{K}_0\rangle = |K_0\rangle \tag{18}$$

Now notice the eigenvalues of C are 1 and -1, and the eigenstates are given by K_1 and K_2 as follows:

Problem 3. Show

$$C|K_1\rangle = -|K_1\rangle, \qquad C|K_2\rangle = |K_2\rangle \tag{19}$$

This shows that anti-particle of K_1 is K_1, and anti-particle of K_2 is K_2.

Then, Gell-Mann and Pais went on to say that if charge conjugation is preserved in the decay, "a possible decay mode for the K_1 is not a possible one for the K_2, and *vice versa*."[1] [2] They further argued "the K_1 must go into a state that is even under charge conjugation, while the K_2 must go into one that is odd. Since the decay modes are different and even mutually exclusive for the K_1 and K_2, their rates of decay must be quite unrelated. There are thus two independent lifetimes, one for the K_1 and one for the K_2."

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[1] Gell-Mann, M. and Pais, A. (1955) *Behavior of Neutral Particles under Charge Conjugation.* Physical Review, 97 (5). pp. 1387-1389.

[2] Italic in original. In their paper, they also wrote ${\theta_1}^0$ and ${\theta_2}^0$ instead of K_1 and K_2 as kaons were called θ.

At the time, only two pions decay mode of K^0 and $\bar{K}^0$ were known. In that paper, Gell-Mann and Pais bravely predicted that there must be another decay mode. According to a memoir published in 1982, "many of the most distinguished theoreticians thought this prediction was really baloney."[3] Nevertheless, in 1956, K_2 was discovered. We now know that the lifetime of K_1 is 8.9×10^{-11} seconds and the one of K_2 is 5.2×10^{-8} seconds; K_2 lives about 600 longer than K_1.

However, in hindsight, Gell-Mann and Pais's reasoning was only partially correct. C is not conserved in weak interactions such as the ones responsible for decay of the neutral kaons. What they should have said was "if CP is conserved in the decay." Let's correct their arguments accordingly. K^0 and $\bar{K}^0$ have odd parity. In other words,

$$P|K^0\rangle = -|K^0\rangle, \qquad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle \tag{20}$$

Thus, combining this relation with (18), we get

$$CP|K^0\rangle = -|\bar{K}^0\rangle, \qquad CP|\bar{K}^0\rangle = -|K^0\rangle \tag{21}$$

which leads to

$$CP|K_1\rangle = |K_1\rangle, \qquad CP|K_2\rangle = -|K_2\rangle \tag{22}$$

If CP is preserved during decay, the decay product of K_1 must be CP even, while the decay product of K_2 must be CP odd. So, let's examine the CP of the decay products. Let's start with 2π. $2\pi^0$ is obviously C even, as π^0 is anti-particle of its own. $\pi^+ + \pi^-$ is C even as well, because C of π^+ is π^- and vice versa. To figure out parity, remember that we have mentioned in the last article that pions have odd parity. Since there are two pions in the decay product, the parity must be even as $(-1)^2 = 1$. Thus, "$\pi^+ + \pi^-$" and "$2\pi^0$" are CP even. So, we have

$$K_1 \to \pi^+ + \pi^- \quad \text{or,} \quad K_1 \to 2\pi^0 \tag{23}$$

Now, let's analyze CP of three pions, namely $\pi^+ + \pi^0 + \pi^-$. C is even for $3\pi^0$ for the obvious reason previously mentioned. $\pi^+ + \pi^0 + \pi^-$ is C even as well, because under C, $\pi^+ + \pi^-$ goes to $\pi^+ + \pi^-$ as mentioned, and π^0 goes to π^0. P is odd, because we have $(-1)^3 = -1$. In conclusion, CP is odd. Thus, we have

$$K_2 \to \pi^+ + \pi^0 + \pi^- \quad \text{or,} \quad K_2 \to 3\pi^0 \tag{24}$$

So, they fit nicely.

All this is based on the assumption that the weak interactions respect CP invariance, which is true but only approximately. As promised in our

[3] J. W. Cronin and M. S. Greenwood, Phys. Today (July 1982), p. 38.

earlier article "CP violation and the 2008 Nobel Prize in Physics," let me conclude this article by briefly commenting on this. As K_2 lives much longer than K_1, K_2 travels farther before being decayed than K_1 does. As both decays are exponential, if we produce K_0 and if it travels certain distance, virtually all that survive must be K_2. ($e^{-600\sharp}$ for a small number $\sharp \sim 1$ is virtually 0.) In 1964, it was reported that after a beam traveled 57 feet, 45 two pions events out of 22700 decay events were detected.[4] This means that CP is slightly violated, as we now have events that go from CP odd to CP even. Thus, CP is violated. The experimenters won the Nobel Prize in 1980.

Summary

- K^0 and $\bar{K}^0$ which have definite strangeness do not are not mass eigenstates. Thus, they do not have definite life-times.
- On the other hand, K_1 and K_2 which are linear combinations of K^0 and $\bar{K}^0$ and CP eigenstates, are approximately eigenstates of mass and have definite life-times.
- CP violation was first detected in neutral Kaon system; CP changing decay was detected.

[4] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay. (1964) *Evidence for the 2π Decay of the ${K_2}^0$ Meson.* Phys. Rev. Lett. 13, 138