Newton's law of universal gravitation

In the last article, we explained why the gravitational force between two objects is inversely proportional to the square of the distance between them. In this article, I explain why the gravitational force between two objects is proportional to the product of the mass of each object.

See Fig. 1. Both the blue ball and the purple ball have the mass m. Their gravitational attraction between them is F. The blue ball attracts the purple ball with the force F and the purple ball attracts the blue ball with the same force F. The forces are denoted by the black arrows.

Figure 1: The gravitational attraction between the blue ball and the purple ball is F.

See Fig. 2. Now, on the left, in addition to the blue ball, there is a red ball, which also has mass m. Then, the purple ball will attract the red ball with the same force as it attracts the blue ball. Similarly, the red ball will attract the purple ball with the same force as the blue ball attracts the purple ball. In other words, the balls on the left attract the blue ball with force $2F$, and the blue ball attracts the balls on the left with the force $2F$. Altogether, the balls on the left have mass $2m$.

Figure 2: The gravitational attraction between the balls on the left and the purple ball is $2F$.

See Fig. 3. Now, on the left side, in addition to the blue ball and the red ball, there is a green ball, which also has mass m. From the same logic, the balls on the left attract the blue ball with the force $3F$, and the blue ball attracts the balls on the left with force $3F$.

In conclusion, in Fig. 1, the object on the left has mass m and the gravitational force between the object on the left and the object on the right is F. In Fig. 2, if we regard the two

Figure 3: The gravitational attraction between the balls on the left and the purple ball is $3F$.

balls as one object, the object on the left has mass $2m$ and the gravitational force between the object on the left and the object on the right is $2F$. In Fig. 3, if we regard the three balls as one object, the object on the left has mass $3m$, and the gravitational force between the object on the left and the object on the right is $3F$.

What do we learn? The gravitational force is proportional to the mass of the object on the left side. When the mass on the left is m, the force is F. When the mass on the left is $2m$, the force is $2F$. When the mass on the left is $3m$, it is $3F$.

Now, we can play the same game by adding balls on the right. Then, the conclusion will be that the gravitational force is proportional to the mass of the object on the right side.

In conclusion, the gravitational force is proportional to the product of the mass on the left and the mass on the right. For example, if there are three balls on the left (i.e., the mass is $3m$) and there are four balls on the right (i.e., the mass is $4m$), the object on the left (i.e., the three balls) will attract the object on the right (i.e., the four balls) with the force $12F$, and vice versa.

Therefore, if an object has mass m_1, and another object has mass m_2, the gravitational attraction between them is proportional to $m_1 m_2$. In the last article, we have seen that the gravitational attraction is proportional to $1/r^2$, if the distance between them is r. Thus, the gravitational attraction between them is proportional to $m_1 m_2/r^2$. If we call the proportionality constant G, we can conclude that the gravvitational attraction between them is given by

$$F = G\frac{m_1 m_2}{r^2} \tag{1}$$

G is called "Newton's constant" or "the gravitational constant" or "the universal gravitation constant" and given by 6.674×10^{-11} $\mathrm{m^3 kg^{-1} s^{-2}}$. See how this number is so small. That is the reason why we do not feel this force in our everyday lives other than the one that the Earth attracts. In other words, as G is so small, it is hard to feel the gravitational force, unless the one attracting us is as heavy as the Earth. You will be able to confirm this by solving the following problem.

Problem 1. Suppose you weigh 60 kg and are 1 meter away from a fat person who weighs 100 kg. What is the gravitational attraction between you and the fat person? Compare this force with the weight of a grain of sand (4.4 mg) or a grain of sugar (0.625 mg) or a grain of salt (0.0585 mg).

However, if we use a very sensitive apparatus, we can actually notice the small gravitational force even in our daily lives. This apparatus is used to detect where oil or certain materials are located beneath the ground. We will have an occasion to talk more about it in our article "Experiments to test equivalence."

Summary

- The gravitational attraction F between two objects is given by

$$F = G\frac{m_1 m_2}{r^2}$$

  where m_1 and m_2 are masses of the object, and r is the distance between them. G is called Newton's constant.