Normal force

See Fig. 1. An object with mass m is on a box with mass M and is not moving. Of course, we know that the downward gravitational force $-mg\hat{k}$ is acting on the object. If there is no other force, the object must fall to the ground. However, it doesn't. Why? Because the box counteracts the gravitational force. In other words, the box exerts an upward force called "normal force" to the object. If we denote this force by $\vec{N}$, the net force (i.e. the total force) applied to the object is given by $\vec{N} - mg\hat{k}$. Since the object is not moving, we know that the net force must be zero. The conclusion is $\vec{N} = mg\hat{k}$. In other words, the normal force is equal to the magnitude of the gravitational force acting on the object, and opposite in its direction. So, is the normal force the reaction of the gravitational force $-mg\hat{k}$? No. The reaction of the gravitational force that the earth acts on the object is the gravitational force that the object acts on the earth. Then, does it mean that there is no action-reaction counterpart of the normal force $\vec{N}$? No. The reaction of $\vec{N}$ is the force that the object acts on the box. It is denoted in the figure as $-\vec{N}$.

From now on, let's think about what forces are acting on the box. We have the gravitational force $-Mg\vec{k}$, and as just mentioned $-\vec{N}$. So, if we add these two forces, they are $-(M+m)g\vec{k}$. But, notice again that the box is not moving. It's because another normal force $\vec{N}'$ is acting on the box by the ground. Since the net force acting on the box i.e. $\vec{N} - (M+m)g\vec{k}$ must be 0, we conclude

$$\vec{N}' = (M+m)g\vec{k} \tag{1}$$

Actually, there is another way of seeing this. If we see the box and the object as one object with mass $M+m$, its gravitational force is $-(M+m)g\vec{k}$, so the normal force must

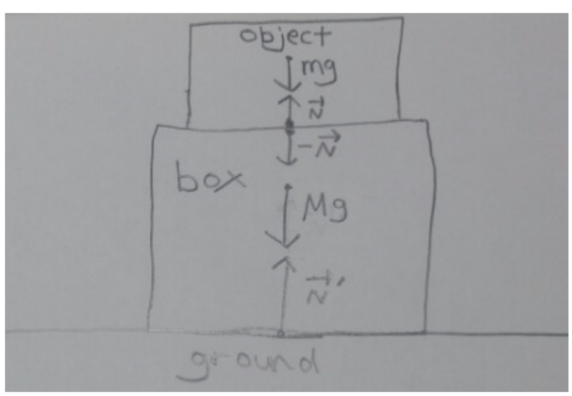

Figure 1: Normal forces on an object and on a box

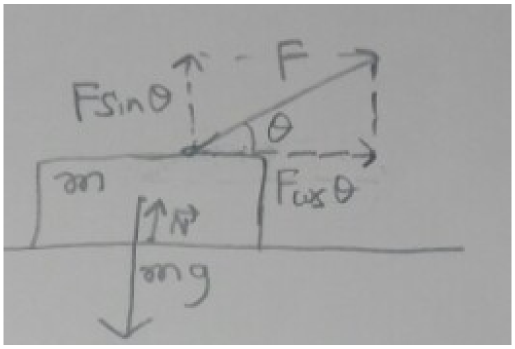

Figure 2: Work done by the normal force

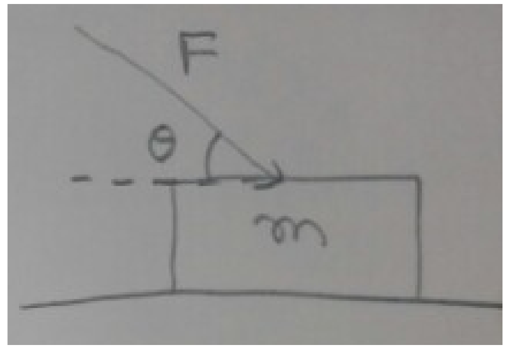

Figure 3: Find the normal force!

be $\vec{N}' = (M+m)g\vec{k}$ as it must be equal to the magnitude of the gravitational force and opposite in its direction.

Now, let's think about how much work a normal force does when an object moves on a ground. See Fig. 2. A force with magnitude F is acting on an object with mass m. The force makes an angle θ with the plane. Here, we will assume that the force is not big enough for the object to fly; it moves along the ground. If it moves along the ground s meters, what is the work done on the object by the gravitational force? We know that the gravitational force $-mg\hat{k}$ is perpendicular to the moving direction. As $\cos 90°$ is 0, the work done by the gravitational force is

$$W_{\text{grav}} = mgs\cos 90° = 0 \tag{2}$$

Problem 1. Explain why the work done by the normal force is also 0 in this case.

As the total work done on the object is the sum of the work done by the gravitational force, the work done by the normal force, the work done by F, we can conclude that the total work done on the object is $Fs\cos\theta$. The normal force and the gravitational force do not contribute.

Now, let's calculate the normal force $\vec{N}$ in this case. The vertical component of the force F is $F\sin\theta$. As we know that the object is not moving along the vertical direction, we know that the net force along vertical direction must be zero. As the gravitational force on the object is given by $-mg\vec{k}$, we obtain

$$\vec{N} + F\sin\theta\hat{k} - mg\vec{k} = 0 \tag{3}$$

We conclude

$$\vec{N} = (mg - F\sin\theta)\vec{k} \tag{4}$$

Problem 2. See Fig. 3. What is the normal force in this case?

Summary

- Normal force is the force exerted from floor when there is an object on the floor. Due to normal force, the weight of the object is balanced, and does not break the floor to fall into the center of the Earth.