Parallax, how do we determine distances to stars?

Parallax is one of the methods astronomers use to determine the distance to stars. We will explain what parallax is in this article.

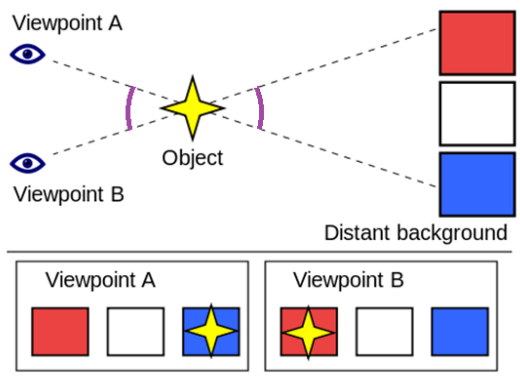

Figure 1: Parallax. [1]

To better visualize it, see Fig. 1. There is a yellow object. From viewpoint A, the object appears to be placed in front of the blue background. On the contrary, from viewpoint B, the object appears to be placed in front of the red background. This difference in the apparent position is called "parallax". This difference can be quantified by either of the two purple angles. It doesn't matter which one as they are equal. Notice that this angle would be smaller if the yellow object were placed farther away from the viewpoints. This means that the parallax is smaller if an object is located farther away. This should be familiar to anyone who has taken a train. If you are on a train and look through a window, the things near you will seem to pass by very fast. On the other hand, things far away from you, such as mountains, will seem to pass by less slowly. This is because the things farther away have less parallax. A more extreme case would be how the Moon appears to us. No matter how fast you move, the Moon seems to be following you always. This is because the Moon is so far away that it is quite impossible for you to notice that the apparent position of the Moon changes as you move. However, the Moon is actually the nearest celestial object, and therefore, among celestial objects, it is the easiest one to detect parallax. If the Moon is seen on two different points on the Earth, the parallax (the one corresponding to the purple angle in Fig. 1) is as large as 2 degrees.

So, how can astronomers use parallax to determine the distance? I already mentioned that the farther the object the smaller parallax is. This means that we can use the parallax to find the distance to the object. If you know trigonometry you will easily figure out how to do so. Actually, even though you don't know trigonometry you can feel the distances to objects by parallax using gut feeling. God gave us two eyes instead of one because he wanted us to feel the distances to objects. Just like in Fig. 1, we can feel the parallax because we have two eyes (i.e., two viewpoints). If you can't believe me, perform a simple experiment. First, hold one pencil on your left hand, and another pencil on your right hand. Then, try to make them closer together and eventually make the tips of the two pencils touch each other. Easy? Now, close one of your eyes and try it again. It won't be easy. When you have two eyes, you can feel the distance to the two pencils, so you can easily make them meet together. However, with only one eye, you lose the sense of distances to the pencils.

Actually, Hipparchus (ca. BC 190∼ca. BC 120), used parallax to measure the distance to the Moon. Of course, as the Moon is moving, to measure its parallax, it would be necessary to look at it at the same time. As the clock was not available then, Hipparchus used the solar eclipse. During the solar eclipse of March 14, 190 BC, the total eclipse was observed around the Hellespont (i.e., the Sun was completely covered by the Moon), while about 4/5 of the solar disc seemed to be covered by the Moon in Alexandria, Egypt. (The distance between them is about 1000 km.) From this data, he concluded that the distance to the Moon is between 71 and 83 if the Earth's radius is 1. The correct value is about 60. Actually, he also determined the distance to the Moon using other data: that the diameter of the shadow of the Earth which can be seen during a lunar eclipse is 2.5 times the size of the Moon. Actually, to obtain the distance to the Moon, he needed to assume the distance to the Sun. As he wouldn't believe that the Sun is so far away, he assumed that the distance to the Sun is 490, the minimum distance of which its parallax wouldn't be noticeable at two points on the Earth. Thus, he obtained that the distance to the Moon was between 62 and $72\frac{2}{3}$. In reality, the distance to the Sun is about 23000, much larger than 490. If he assumed that the distance to the Sun is practically infinity, he would have gotten 59, a very good value.

Problem 1. Let's obtain this value. Hipparchus used the fact that the Sun and the Moon have the same apparent size, which is 1/650 of 360°. See Fig. 2.

S is the center of the Sun, O the center of the Earth, and M the center of the Moon. Therefore, $\overline{OM}$ is the distance to the Moon, which is approximately equal to $\overline{OB}$ as the angle $\angle BOC$ is very small. $\overline{BC}$ is the shadow of the Earth, which is 2.5 times the diameter of the Moon. First, show that $\angle BAO + \angle ABO = \angle AOS + \angle BOM$. Now, we will assume that the Sun is so far away that its parallax measured on the Earth is negligible. In other words, $\angle BAO \approx 0$. (On the other hand, $\angle AOS$ is not negligible, because the radius of the Sun is much larger than that of the Earth.) Using these facts, obtain $\angle ABO$. Now, we can obtain the distance $\overline{OB}$ by making an approximation that $\angle ABO$ is very small. If $\angle ABO = \theta$ in radians, $\overline{OB}$ is given by $1/\theta$ assuming that the radius of the Earth is 1.

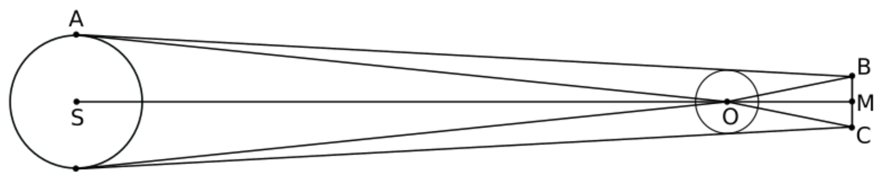

Figure 2: Schematic representation of the Sun-Earth-Moon system for Problem 1.

Now, let's continue with annual parallax. If the Earth revolves around the Sun, it means that the Earth's position is changing. Therefore, if we see stars on different seasons, their apparent positions must seem to be different. Of course, this is unless the stars are too far away. In such case, the difference in the apparent positions would be too small. This is the concept of annual parallax[1]. Pay attention to Fig. 3. The black dots on the right are very distant stars, whose apparent positions hardly change. They serve as background to check the apparent position change of the near star.

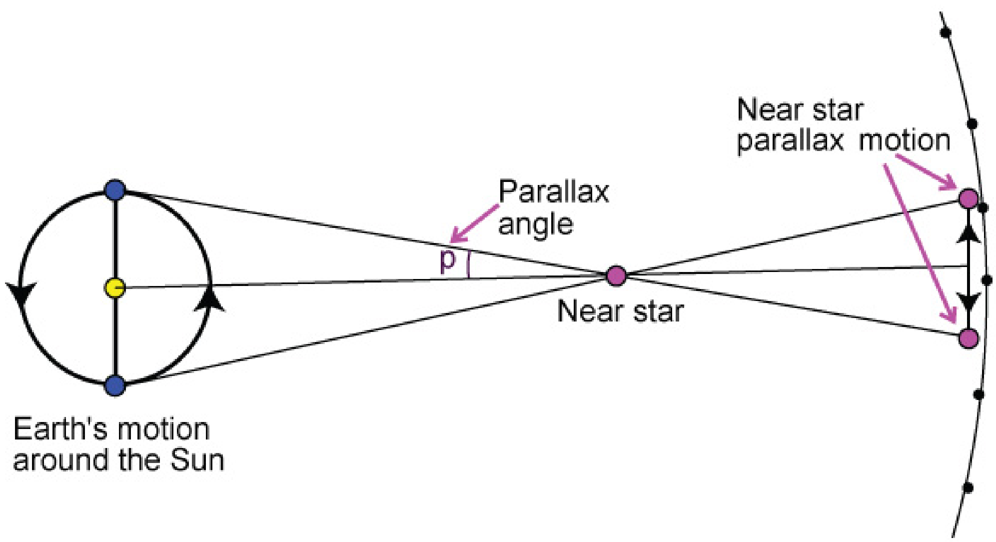

Figure 3: Same as in Fig. 1 but for annual parallax.

Notice that astronomers use a different convention for the annual parallax than the others. The annual parallax is defined by half of the one we used in Fig. 1. However, as all the stars are so far away compared to our Solar system, the annual parallaxes are so small. Thus, the annual parallax was first detected only in 1838 with better technology; Friedrich Bessel detected that the star called "61 Cygni" had a parallax of 0.314 arcseconds (1 arcminute is

[1]The word "annual" means "yearly" and comes from the Latin word *annus* which means "year."

60 arcseconds). The modern value is 0.286 arcsec. Before the invention of telescopes, the best resolution with naked eyes was around 1 arcminute (=1/60 degree), achieved by Tycho Brahe in the 16th century. So, the annual parallax is really really small.

Problem 2. Let's say that you see an ant 10 km away. How much does the ant have to move for its apparent position to change about 0.3 arcsecond (i.e., the annual parallax of 61 Cygni)?

When I learned the annual parallax for the first time, I learned that the distance to the star is inversely proportional to the annual parallax. I thought that that couldn't be true. Let's see why I thought so.

Problem 3. See Fig. 3 again. Let's say the radius of the orbit of Earth is R. Express the distance from the Sun (denoted by a yellow ball) to the near star (denoted by purple) by using R and *a trigonometric function* of the angle p.

Luckily, I learned later that $\tan\theta \approx \theta$, and my doubt was completely resolved. In "Asymptotic behavior of polynomials," you will be invited to check how good approximation it is. Currently, the annual parallax of 61 Cygni can be measured within about 0.02% (i.e., 1 part in 5000) of accuracy. You will check that, for $\tan\theta \approx \theta$ not to be a good enough approximation to determine the distance to 61 Cygni, we would need to determine the annual parallax of 61 Cygni at least more accurately than 1 part in 10^{12}, which is totally impossible with modern technology, as it requires 200 million ($= 10^{12} \div 5000$) times of better accuracy.

In "History of Astronomy from the late 17th century to the early 20th century," we will introduce new methods to determine the distances in cases when the annual parallax is too small to notice even with modern technologies.

Summary

- Parallax is the difference in the apparent position of an object, due to the different positions of the observers. Parallax is measured by an angle, and the farther the object, the smaller the parallax. Parallax can be used to determine the distance to the object.

- Annual parallax is the change in the apparent position of stars, due to the motion of the Earth around the Sun.

- Astronomers use the annual parallax to determine the distances to stars if they are not too far away to notice the annual parallax.

References

[1] Adapted from `https://commons.wikimedia.org/wiki/File:Parallax_Example.svg`.