A short introduction to quantum mechanics II: revisiting double slit experiment

Let's consider the double slit experiment again as advertised in our earlier article "De Broglie's matter waves." Let's call the wave function of the particle that goes through the first slit and reaches a certain point of the screen labeled by "x" $\phi_1(x)$, and the one of the particle that goes through the second slit and reaches the same point of the screen $\phi_2(x)$. Remembering that in our earlier article "Young's interference experiment" we explained that the displacement of two waves reaching the same point of the screen is added, which results in the interference pattern, the wave function $\phi(x)$ of the particle that goes through the both slits would be $\phi(x) = \phi_1(x) + \phi_2(x)$. For example, if $\phi(x) = 0$, it corresponds to the destructive interference. Remembering that the probability that the particle will be found is proportional to the square of the corresponding coefficients of the wave function (in our case the coefficient is simply $\phi(x)$), we see that the probability that the particle will be found at the position x on the screen is proportional to the following quantity: $|\phi_1(x) + \phi_2(x)|^2$.

However, if you place a detector and check which slit the particles goes through, the wave function becomes either $\phi_1(x)$ or $\phi_2(x)$ upon measurement. In this case, the probability that the particle that passed through the first slit will be found at x is proportional to $|\phi_1(x)|^2$. Similarly, the probability that the particle that passed through the second slit will be found at x is $|\phi_2(x)|^2$. So, the probability that the particle that passed through either the first slit or the second slit will be found at x is proportional to $|\phi_1(x)|^2 + |\phi_2(x)|^2$. Notice that this is different from $|\phi_1(x) + \phi_2(x)|^2$, which is the probability that the particle that passed through *both* the first slit *and* the second slit will be found at position x. Quantum mechanics is indeed interesting, as a single particle must go through the two slits at the same time.

Summary

- The probability related to the wave function of a particle that goes through either the first slit or the second slit is given by $|\phi_1|^2 + |\phi_2|^2$.
- The probability related to the wave function of a particle that goes through *both* the first slit *and* the second slit is given by $|\phi_1 + \phi_2|^2$.

- Quantum mechanics is against our macroscopic intuition, as $|\phi_1 + \phi_2|^2 \neq |\phi_1|^2 + |\phi_2|^2$, which implies that a single particle must go through the two slits at the same time.