Quotient rule

In this article, we will investigate how the derivatives of the following form of functions can be obtained:

$$\frac{f(x)}{g(x)} \tag{1}$$

To this end, let $h = f/g$ and try to find the derivatives of h in terms of f, g, f' and g'. As $f = gh$, we have:

$$f' = g'h + gh' \tag{2}$$

which imply

$$\left(\frac{f}{g}\right)' = h' = \frac{f' - g'h}{g} = \frac{f' - g'(f/g)}{g} = \frac{f'g - fg'}{g^2} \tag{3}$$

This is the quotient rule.

Problem 1. Let's derive the quotient rule by an alternative way. To this end, let's first calculate the derivative of $(1/g(x))$. Using chain rule, we have:

$$\left(\frac{1}{g(x)}\right)' = -\frac{1}{g(x)^2} \cdot g'(x) \tag{4}$$

Given this, let's calculate the derivative of (1). Let, $j = 1/g$. Then, we have:

$$\left(\frac{f}{g}\right)' = (fj)' \tag{5}$$

Now, use Leibniz' rule to prove (3). (Hint[1])

Problem 2. Show

$$(\tan x)' = \frac{1}{\cos^2 x}$$

Problem 3. Calculate the followings.

$$\left(\frac{x+2}{x^2+3}\right)' = ?, \qquad \left(\frac{x^2}{2x^2+1}\right)' = ?, \qquad \left(\frac{\sin x}{x^2}\right)' = ? \tag{6}$$

Summary

- The quotient rule is given by

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

[1]Use $j = 1/g$ and j' given in (4)