Simple pendulum

Consider the pendulum in the left one of Fig.1. Suppose you gently release the ball with mass m. Then, it will oscillate. In this article, we will find the period of this oscillation. Let's say that the length of string holding the ball is l, the mass of string is negligible, and the gravitational constant is g, and consider the moment when the string attached to the ball makes an angle of θ with the vertical line. There are two forces acting on the ball. The tension of string and the gravitational force. Given this, notice that it can only move along the perpendicular direction of the string, since the length of string is always constant. (i.e. it cannot move along the direction of string) Therefore, to find the differential equation of θ we need to only consider the component of force along the arc. The position of the ball on the arc is given by $s = l\theta$. (See the left figure.) Also, the component of the force along the arc is $mg\sin\theta$. (See the right figure.) Therefore, from Newton's second law, we can write the following equation:

$$m\frac{d^2s}{dt^2} = -mg\sin\theta \tag{1}$$

$$ml\frac{d^2\theta}{dt^2} = -mg\sin\theta \tag{2}$$

For a small θ, the above equation can be approximated as:

$$l\frac{d^2\theta}{dt^2} = -g\theta \tag{3}$$

So, we have the exact same form of the equation of motion as the one we saw in our earlier article "Harmonic oscillator." Only the coefficient is different. Therefore we can use

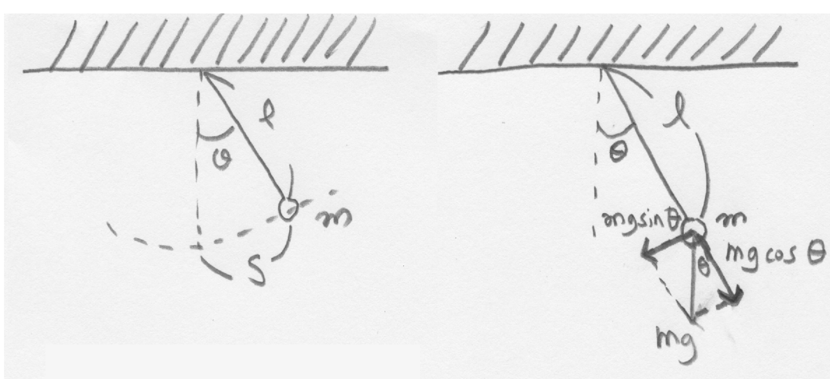

Figure 1: pendulum

the earlier formula for the period of harmonic oscillator there. We get:

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{4}$$

Of course, this formula is not exact as we used the approximation $\theta \approx \sin\theta$. Nevertheless, according to Wikipedia, the formula is exact within 1% even when the maximum angle (i.e. amplitude) is as big as $20°$.

Summary

- Simple pendulum can be regarded as an harmonic oscillator, when the swinging angle θ is small enough to be made an approximation $\sin\theta \approx \theta$.