The derivation of Snell's law using calculus

In this article, you will be asked to derive Snell's law from Fermat's principle. Please first read our earlier article "Fermat's principle and the consistency of physics."

See Fig. 1. Light will go from point A to point B. The light will enter water at point C. You will find out the position of C, denoted here as x, which minimizes the time that the light takes to travel from point A to point B. To this end, we need to first find out this time in terms of x. We will give you a guide-line and you will find out the answer. We will assume that the speed of light in air is v_1 and the speed of light in water is v_2.

Problem 1. What is the distance between the point A and point C in terms of x? How about the distance between the point C and B?

Problem 2. How long does it take to travel from the point A and the point B in terms of x?

Problem 3. To find the minimum time, you need to differentiate the traveling time with respect to x and set it equal to 0. Obtain an equation by doing so, but do not actually solve the equation. If you do it correctly, you will obtain:

$$\frac{1}{v_1}\frac{x}{\sqrt{h_1^2+x^2}} = \frac{1}{v_2}\frac{L-x}{\sqrt{h^2+(L-x)^2}} \tag{1}$$

Problem 4. Express $\sin\theta_1$ in terms of h_1 and x. Express $\sin\theta_2$ in terms of L, x, and h_2.

Problem 5. Express (1) in terms of $\sin\theta_1$ and $\sin\theta_2$ using the result of Problem 4. You

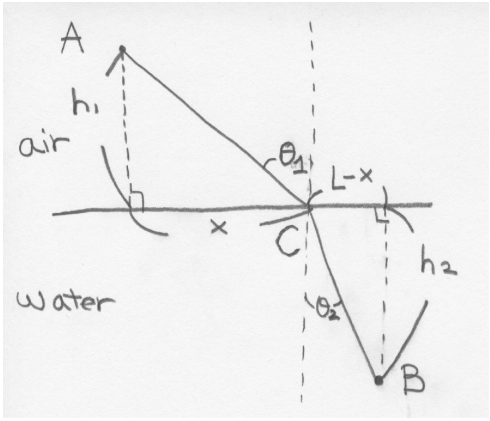

Figure 1: Light traveling from A to B

will get the following Snell's law:

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} \tag{2}$$