Specific heats of gases

Suppose you have a container with volume V. Inside it, you have a gas with n moles of molecules with initial temperature T_i. How much energy is needed to raise the temperature of the gas by ΔT? Of course it depends on gases.

Let's first consider the case that the gas concerned is monatomic. The initial internal energy U_i of the gas is given by:

$$U_i = (nN_A)\frac{3}{2}kT_i = \frac{3}{2}nRT_i \tag{1}$$

Similarly, the final energy E_f of the gas is given by:

$$U_f = \frac{3}{2}nR(T_i + \Delta T) \tag{2}$$

Therefore, the energy required is given as follows:

$$\Delta Q = U_f - U_i = \frac{3}{2}nR\Delta T \tag{3}$$

Therefore, to raise the temperature of monatomic gas of 1 mole by 1 Kelvin at a constant volume, we need $\frac{3}{2}R$ Joules of energy. This is denoted by C_v and called "the molar specific heat at constant volume." In other words, for monatomic gas, we have:

$$C_v = \frac{3}{2}R \tag{4}$$

It is left as an exercise to the readers to show that the molar specific heat at constant volume for diatomic gas is given by $\frac{5}{2}R$. In any case, if we denote the change of the internal energy of the gas by ΔU, it is easy to see that we have the following equation:

$$\Delta U = nC_v\Delta T \tag{5}$$

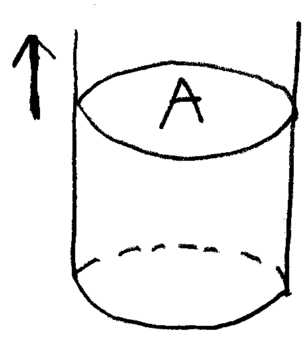

Figure 1: gas inside a piston

Now, we will consider the case in which the volume of the gas is not constant; instead of putting the gas inside a fixed container, we will put it inside a piston, of which the upper cover can move freely. In this way, the gas can either expand or contract, changing its volume. See Fig.1. We see that the area of the cover is given by A. Suppose now, the volume of the gas changes by ΔV, then it is easy to see that the cover will move upward by $\Delta V/A$. Also, the upward force on the cover due to the pressure of the gas is given by PA, if the pressure of the gas is P. Now, if this force moves the cover upward as the volume of the gas expands by ΔV, the work done on the cover can be calculated as follows:

$$W = F\Delta s = (PA)(\Delta V/A) = P\Delta V \tag{6}$$

Given this, if you heat a gas by energy ΔQ, some will be used to do work and the other will be used to raise U the internal energy of the gas (i.e. the kinetic energy of the gas molecules). This relation can be expressed mathematically as follows:

$$\Delta Q = P\Delta V + \Delta U \tag{7}$$

In our earlier case of constant volume, we had $\Delta V = 0$.

Now, we are ready to calculate C_p, the molar specific heat of gas at constant pressure. If the initial volume and the initial temperature of a gas is V_i, and T_i, we have:

$$PV_i = nRT_i \tag{8}$$

And if the gas expands by ΔV, and the temperature increases by ΔT, we have:

$$P(V_i + \Delta V) = nR(T_i + \Delta T) \tag{9}$$

Subtracting (8) from (9), we obtain:

$$P\Delta V = nR\Delta T \tag{10}$$

Using this equation along with (7) and (5), we finally obtain:

$$\Delta Q = nC_p\Delta T = nR\Delta T + nC_v\Delta T \tag{11}$$

$$C_p = C_v + R \tag{12}$$

For example, C_p for monatomic gas is $\frac{5}{2}R$ and for diatomic gas is $\frac{7}{2}R$.

Summary

- Specific heat for constant pressure is bigger than specific heat for constant volume, as in the former case the gas does work because it's expanding while in the latter case the gas doesn't work because it's at constant volume. More specifically, we have $C_p = C_v + R$.
- C_v for monatomic gas is $\frac{3}{2}R$.