Specific heat and latent heat

In our last article, we explained what heat capacity and specific heat are. In this article, we will put them into mathematical symbols, and additionally introduce latent heat.

Heat capacity C is given by the amount of heat needed to raise the temperature of an object by 1 degree. In other words, to raise the temperature of the object by ΔT degrees, the heat we need is given by

$$Q = C\Delta T \tag{1}$$

As the specific heat is the heat capacity per unit mass, we have

$$c = \frac{C}{m} \tag{2}$$

where m is the mass of the object. In other words, we have

$$Q = mc\Delta T \tag{3}$$

Now, suppose you have two objects made out of the same material. Let's say that object 1 has mass m_1, temperature T_1, and object 2 has mass m_2 temperature T_2. If you bring them into thermal contact, the final temperature T will be the weighted average of two temperatures i.e.,

$$T = \frac{m_1T_1 + m_2T_2}{m_1 + m_2} \tag{4}$$

Problem 1. Check that the above formula becomes just the usual average, when m_1 is equal to m_2.

Now, suppose they have different specific heats: c_1 and c_2 respectively. Then, the "weight" is no longer just m, but $C = mc$, the heat capacity. Thus, the final temperature becomes

$$T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2} \tag{5}$$

(**Problem 2.** Check that (5) reduces to (4) when c_1 is equal to c_2.)

Now, let's derive this formula somewhat more rigorously. Let's say $T_1 < T_2$. The case when $T_1 > T_2$ is similar. Then, the heat object 1 gained is the heat object 2 lost. (Heat is conserved in this case, because nobody is heating or rubbing with friction.) Let's call this heat Q, and the final temperature T. Then, applying (3) to object 1, we have

$$Q = m_1 c_1 (T - T_1) \tag{6}$$

As losing Q of heat is same as gaining $-Q$ of heat. Thus, upon applying (3) to object 2, we have

$$-Q = m_2 c_2 (T - T_2) \tag{7}$$

Adding (6) and (7), we get

$$m_1 c_1 (T - T_1) + m_2 c_2 (T - T_2) = 0 \tag{8}$$

$$m_1 c_1 T + m_2 c_2 T = m_1 c_1 T_1 + m_2 c_2 T_2 \tag{9}$$

Thus, we exactly get (5).

(**Problem 3.** Show that you get the same conclusion when $T_1 < T_2$.)

Let us now explain the latent heat. Ice is colder than water. If you warm ice, it becomes water (i.e., melts) at 0°C. This temperature is known as "freezing point." Vapor is hotter than water. If you warm water, it becomes vapor (i.e., boils) at 100°C. This temperature is known as "boiling point."

So, water has three phases: solid, liquid, and gas. If you warm water or ice or vapor, the temperature usually rises. However, when it changes the phase, the temperature doesn't rise, because the heat is used to change the phase. This is known as "latent heat." Of course, the latent heat is proportional to the mass of object changing its phase. Just like heat capacity per unit mass is called specific heat, latent heat per unit mass is called "specific latent heat" L. For example, it takes the following amount of heat

$$Q = Lm \tag{10}$$

for a material with m to change its phase. For example, for vaporization of water L is given by 540 cal/g. Then, to change 5 g of 100°C water into vapor, we need 2700 cal (=540 cal/g×5 g).

Now, let's apply the concept of latent heat to solve a problem. Let's say there is 50 g of vapor with 100°C in an insulated box (i.e., heat can neither enter or go out from this box). If you put 270 g of water with 80°C into the box, how much vapor will become water? The specific heat of water is 1 cal/g·°C. So, for the 27 g of water to become 100° C, we need

$$Q = 270 \text{ g} \times 1 \text{ cal/g}\cdot{}^\circ\text{C} \times (100^\circ\text{C} - 80^\circ\text{C}) = 5400 \text{ cal} \tag{11}$$

With 5400 cal, we can liquidiate 10 g of water. So, the answer is 10 g. As the initial amount of vapor was 50 g, there will be still 40 g of vapor remaining. If the initial amount of vapor was less than 10 g, all will become water and the final temperature of the water will be less than 100°C because the 80°C water can still cool down even after all the vapor becomes water.

Problem 4. It requires 80 cal to melt 1 g of ice (i.e., L=80 cal/g). With 20 g of 10°C water, how much ice can you melt?

Summary

- If we warm an object with mass m, and specific heat c by heat Q, the temperature risen ΔT satisfies the following relation.

$$Q = mc\Delta T$$

- When a material changes its phase its temperature doesn't rise. Solid, liquid, and gas are examples of phases. The heat required to change the phase is called "latent heat." The latent heat per unit mass is often denoted by L. Thus, if we denote latent heat by Q, we have

$$Q = Lm$$