Standing wave

Consider a string, such as a guitar string, stretched between two clamps and made to oscillate. See Fig.1. It will surely make a sound. What would be the note? This is the question we will answer in this article. First, I want to note that the string can oscillate like a sine wave, even though it moves neither in positive x-direction nor in negative x-direction and stays there. It is called “standing wave.” If it doesn’t oscillate like a sine wave, we can use Fourier series, and make the oscillating motion of the string as a sum of modes of sine waves. Given this, notice that the fact that the both ends are fixed (i.e. displacements zero) strongly constrains the form of the motion of string. Three possible modes of the sine waves are shown in the figure. In the first case, L is half of the wavelength, as the distance between the neighboring points with zero displacement is half of the wavelength. Therefore, we have $\lambda_1 = 2L$. In the second case, L is the wavelength. In other words, $\lambda_2 = L$. In the third case, $\lambda_3 = 2L/3$. It is easy to see that $\lambda_n = 2L/n$ for nth mode, as $n(\lambda/2) = L$ for positive integer n, only positive integer number of half wavelength can fit into L.

Now, what is the note? If v is the speed of traveling waves on the string, we have:

$$v = \frac{\lambda}{T} = \lambda f \tag{1}$$

Therefore, if we denote the frequency of the nth mode by f_n, we have

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} \tag{2}$$

where $n = 1, 2, 3.....$ This is the note that we will hear.

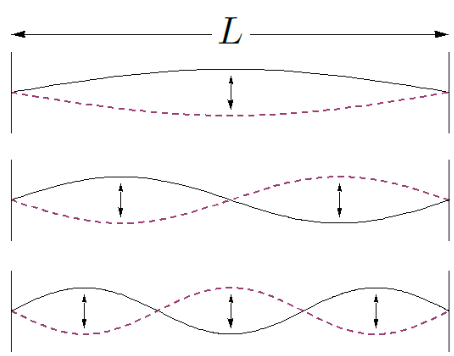

Figure 1: Oscillating strings

Summary

- If two ends of a string is fixed, and the distance between them is L, then as the string vibrates, the wave is not moving. Such a wave is called "standing wave."
- Then, λ, the allowed wavelength of the standing wave satisfies $\lambda = 2L/n$ for a positive integer n.