Surprises in math, the reason why I study physics, and a recommendation for “The Number Devil.”

Suppose you draw an altitude of a triangle as below. The altitude starts from one of the vertices of the triangle and ends at the edge opposite the vertex, forming a right angle with it.

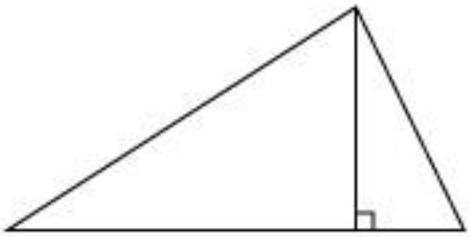

Fig. 1

Actually, there are three vertices and three edges in a triangle. Therefore, you can draw three different altitudes. Suppose you draw them as below.

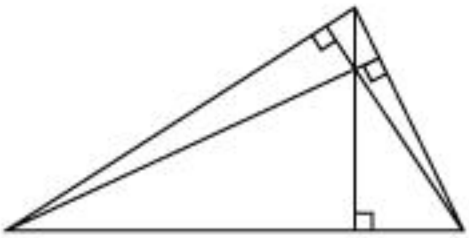

Fig. 2

Perhaps you are surprised to find that the three altitudes intersect at one point. Is this a coincidence? Curious, you try this with other triangles.

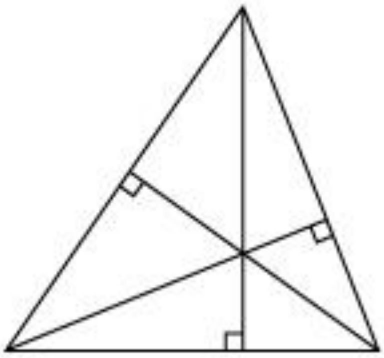

Fig. 3

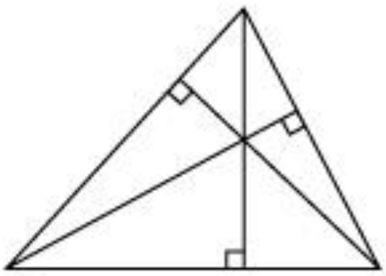

Fig. 4

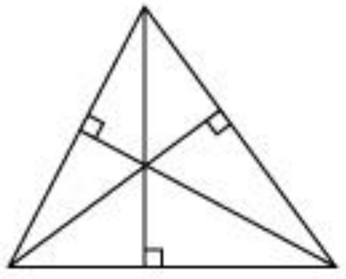

Fig. 5

More surprised, you find that all of the triangles you have drawn satisfy this property. Your interest thus aroused, you may wonder why all the triangles satisfy this property, and be motivated to learn the reason. Actually, it can be proven that every possible triangle satisfies this property. (If you are interested in the proof you can read our later article “The orthocenter of a triangle.”) In fact, the example here surprised Albert Einstein when he was 12 years old.[1]

The more I study physics and math, the more often I encounter such surprises. That’s why I study physics and math: to find the mechanisms behind such surprises. If you watch a magic show that really surprises you, don’t you want to know how the magician did it? I want to know how God did the magic show called “physics.” Besides physics, I also want to know how the magic show called “math” works, especially because the more I study physics, the more I am convinced that the magician called “God” is not only good at mathematical calculations but also knows deep connections between the different branches of mathematics and the difficult theorems which, for mortals, are hard to prove.

Let me tell you an example. In November 1978, John Mckay, an English mathematician who specializes in group theory, was taking a break by reading a paper in number theory. There, he found the following formula.

$$J(\tau) = \frac{1}{q} + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

He was surprised when he found here “196884” a somewhat familiar number. As a group theorist, he knew that 196883 is the dimension of the second smallest irreducible representation of the Monster group (Let’s call this number r2), but he had no idea why a similar number pops out in the number theory. It was hard to believe that this was a coincidence. Thus, he wrote John Thompson, a renowned American Fields Medalist. (Fields Medal is the Nobel prize equivalent of mathematics.) Thompson checked other numbers in

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[1] In his autobiographical notes, Albert Einstein wrote:

“At the age of 12 I experienced a second wonder of a totally different nature: in a little book dealing with Euclidean plane geometry, which came into my hands at the beginning of a schoolyear. Here were assertions, as for example the intersection of the three altitudes of a triangle in one point, which –though by no means evident- could nevertheless be proved with such certainty that any doubt appeared to be out of the question. This lucidity and certainty made an indescribable impression upon me.” –“Albert Einstein Philosopher-Scientist" by Paul Arthur Schilppthe above formula and found out that the other numbers showed similar coincidences. Let me explain what the coincidences are. 1 is the dimension of the smallest irreducible representation of the Monster group, (Let’s call it r1) and 21296876 is the dimension of the third smallest irreducible representation of the Monster group (Let’s call it r3). Then, we have 1=r1, 196884=r1+r2, 21493760=r1+r2+r3 and so on.

Thompson talked about this with an English mathematician John Conway, who formalized all these ideas in more concrete details by working with his former Ph.D. student Simon Norton. In 1979, Conway and Norton proposed what is now called “Conway-Norton conjecture,” which suggested a mysterious connection between the Monster and the J function. Conway named this connection “Moonshine,” because “the stuff we were getting was not supported by logical argument.” In 1992, Richard Borcherds, a British mathematician proved Conway-Norton conjecture using string theory-inspired mathematical theorem among many other theorems. Borcherds won the Fields Medal.

It is my belief that young students must encounter surprises in math and physics so that they are motivated to study them. After all, other than the curiosity to understand surprises, there are no other reasons why pure mathematicians and pure physicists study them. Thus, only when the curiosity to understand such surprises is encouraged, can physics and math education be successful. However, neither the education a century ago nor the current education seems to do so. Einstein said, “It is a miracle that curiosity survives formal education.”

In light of this, I recommend “The Number Devil: A Mathematical Adventure” by Hans Magnus Enzensberger for elementary school or middle school students. It is full of surprising mathematical examples that motivate them to learn mathematics.