By how much does time go more slowly at a lower place?

In our earlier article "Why time goes more slowly at a lower place and what a black hole is," I explained why time goes more slowly at a lower place. The idea is that photons lose energy as they travel upward, which implies that their frequency is also lowered. From this picture, we will indeed derive how much time goes more slowly at a lower place in Section 23 of our later article "An Introduction to General Relativity." However, in this article, we will approach the problem slightly differently. From the Doppler effect, we will show that the frequency of the photons is lowered, as they climb upward. Of course, these two different approaches lead to the same result.

To interpret this phenomenon as a Doppler effect, it is necessary to use the equivalence principle. We will consider a frame with gravitational field as a frame without any gravitational field, but accelerating upwards with the acceleration g. See Fig. 1. When $t = 0$, the frame (or, you can call it an "elevator") is at rest, but it starts to accelerate upwards. Also, at this moment, we shoot a beam of light with frequency f_1 upward from the floor of the elevator. The beam is denoted by the yellow arrow. Assuming that the vertical length of the elevator is h, when will the beam arrive at the ceiling of the elevator? Note that the elevator is moving much more slowly than the speed of light. Therefore, when calculating the time it took for the light to reach the ceiling, we can safely ignore the distance the elevator moved during this time, as it is much smaller than h. This distance is marked by a green arrow in the figure. Therefore, at $t \approx h/c$, the light reaches the ceiling.

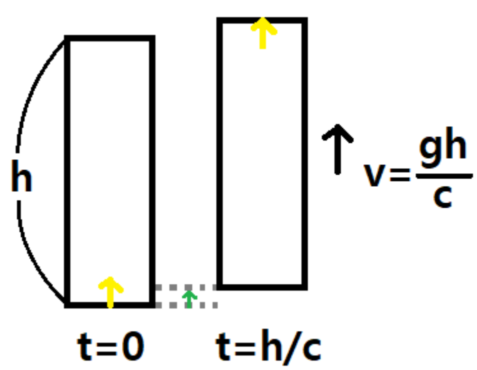

Figure 1: moving frame

What is the speed of the elevator when the light reaches the ceiling? It is given by

$$v \approx g\left(\frac{h}{c} - 0\right) = \frac{gh}{c} \tag{1}$$

Problem 1. Using the result of our earlie article "Doppler effect" show that f_2, the observed freqeuncy of the light detected at the ceiling of the elevator is approximately given by

$$f_2 \approx \left(1 - \frac{gh}{c^2}\right) f_1 \tag{2}$$

Given this, what is t_2, the time elapsed at the ceiling of the elevator while t_1 elapses at the bottom? As we have

$$\frac{t_2}{t_1} = \frac{1/f_2}{1/f_1} \tag{3}$$

we obtain

$$t_2 \approx \left(1 + \frac{gh}{c^2}\right) t_1 \tag{4}$$

This is our final result.

Problem 2. The height of Mount Everest is 8848 meter. How much time elapses at the summit of Mt. Everest while 100 years elpases at the sea level?

Problem 3. Consider the same problem as in Fig. 1, but the case in which the elevator is already moving upward at a speed v_0 when $t = 0$. Assuming v_0 is much smaller than c, show that we get the same conclusion as in (4).

Summary

- We can approach the problem of how much time goes more slowly at a lower place quantitatively by converting a frame with a gravitational field to an accelerating frame without gravitational field and considering the Doppler effect due to the movement of the accelerating frame.