

Twin paradox

Suppose you are at rest, and you see a moving clock. You will see that it ticktocks at slower rate than your own. However, from the point of view of the moving clock, it is at rest itself, and you are the one moving. Therefore, the moving clock will see that your clock ticktocks at slower rate. This seems paradoxical since one of the two clocks, not both, should ticktock at a slower rate than the other one.

However, this is not paradoxical. Since the two clocks are moving apart, there is no way to compare them fairly. If you want to compare them fairly, the moving clock and you have to meet again at a certain place, after the moving clock passed you away initially. In this way, you can compare how much time each of the trip took. However, to do this, at least one of the two must change its velocity so that it can return. Then, the one changing its velocity is not in the inertial frame. Notice also that a passenger inside a spaceship will know whether the spaceship is changing its velocity, even if there is no window in the spaceship. If it changes the velocity, the passenger inside will feel the inertial force. If it doesn't, the passenger inside won't feel the inertial force. So the point of view of the one not changing its velocity and the point of view of the one changing its velocity are not equivalent. The point of view of the one not moving is more "objective."

Therefore, if John, a twin brother of Michael, took a trip from Earth to a distant star and came back, while Michael stayed in Earth, Michael's point of view is objective and John would be younger than Michael, since John's clock ticktocked at a slower rate than Michael's.

Actually, they can track whose time goes more clearly by exchanging light signals every year. This will clarify what actually happened. The result is unambiguous. John would be younger than Michael by the factor $1/\sqrt{1 - v^2/c^2}$. If you are interested in this analysis, check out our later article "Doppler effect and twin paradox revisiting." In any case, time dilation in twin paradox type experiment is successfully confirmed. According to Wikipedia, in 1971, American scientists Hafele and Keating took atomic clocks aboard on commercial airline. They flew them twice around the world, first eastward, then westward, and compared the time elapse with the one that remained at the ground. For the westward one, the clock gained 273 ± 7 nanoseconds, and for the eastward one, the clock lost 59 ± 10 nanoseconds, while the theory of relativity predicted 275 ± 21 nanoseconds and 40 ± 23 nanoseconds for each. (1 nanosecond is 10^{-9} seconds.) So, theory of relativity is indeed quantitatively confirmed.

Let me conclude this article with a historical remark. After Lorentz transformation was derived by Lorentz, and before Einstein came up with theory of relativity, one of the greatest 19th-20th century mathematicians Poincaré noted the problem that we considered in this article: Each observer will see that other's clock would ticktock at slower rate than his or

her own. He could see this as the inverse Lorentz transformation is in the same form as the original Lorentz transformation, with the sign of velocity switched. (Remember our earlier article “Lorentz transformation.”) However, Poincaré thought that this observation is nonsense, and regarded the inverse Lorentz transformation as mere mathematical artifacts without any physical relevance. Now, we know that his thought was wrong.

Summary

- Each of two clocks which move relative to each other at a constant velocity will observe that the other clock will tick more slowly because the other one is moving, while its own is not moving. Both points of view are equivalent and valid.
- However, this is not a contradiction; to compare their time, they have to meet each other. To do so, at least one of them has to change the velocity. Then, their situations are not equivalent.
- The point of view of the one who stayed is more “objective” than the one who turned away to meet the former. The time of the one who turned away elapsed more.