Work and kinetic energy

Surely, I don't doubt that you are familiar with the words "energy" and "work" even though you might have never studied physics. In this article, we will learn how these words are used in physics. However, concerning energy, we will focus only on kinetic energy in this article, and postpone other kinds of energy to later articles.

See Fig.1. Suppose you have an object at rest on the frictionless ground, and you exert force F for the distance s. Certainly, the bigger the force, the more work you have done. Certainly, the longer the distance, the more work you have done. This suggests that the work you have done is given by the following formula:

$$W = Fs \tag{1}$$

Given this, I will now introduce kinetic energy. (The word "kinetic" comes from a Greek word meaning "moving.") It is natural to imagine that the faster an object moves, the more the object's kinetic energy. Actually, the kinetic energy increases by the amount of the work done on the object. Now, we can calculate the kinetic energy in terms of the mass "m" and the velocity "v" of the object as follows.

Since the object was initially at rest, the kinetic energy of the object is zero. Now, observe following. The acceleration of the object is given by $a = F/m$. Furthermore, as the final velocity is v (i.e. the velocity after the object accelerated while moving by the distance s), from our earlier article "Constant acceleration in 1-dimension," we have the following equation:

$$s = \frac{v^2 - 0^2}{2a} = \frac{v^2}{2a} \tag{2}$$

as the initial velocity is zero. Now, E_k, the final kinetic energy must be given by Fs since the initial kinetic energy is zero. So, we get:

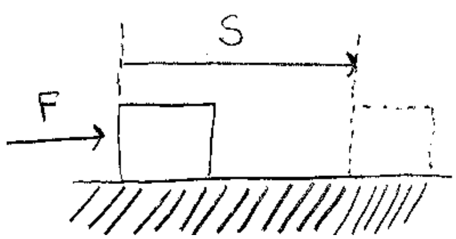

Figure 1:

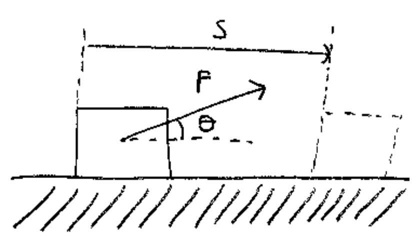

Figure 2:

$$E_k = Fs = F\frac{v^2}{2a} = \frac{1}{2}\frac{F}{a}v^2 = \frac{1}{2}mv^2 \tag{3}$$

This is the kinetic energy of an object with mass m and velocity v.

Given this, let's perform a consistency check. Consider the same situation, but instead of assuming that the initial velocity was 0, let's assume that it is given by v_i, and the final velocity by v_f. Then the change of kinetic energy is given by:

$$\Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \tag{4}$$

while the work done on the object is equal to:

$$W = Fs = F\frac{v_f^2 - v_i^2}{2a} = \frac{1}{2}\frac{F}{a}(v_f^2 - v_i^2) = \frac{1}{2}m(v_f^2 - v_i^2) \tag{5}$$

So, we indeed see that the change in kinetic energy is equal to the work done on the object.

Now, we will discuss a slightly different point. See Fig.2. Suppose now you exert the force F in the direction separated by angle θ from the moving direction, and move the object by the distance s. Certainly, the component of the force perpendicular to the moving direction does not contribute to the acceleration of the object; only the parallel component, which is $F\cos\theta$, contributes. Therefore, the work done is equal to:

$$W = (F\cos\theta)s = Fs\cos\theta \tag{6}$$

If you are not sure about this, let's think about this in this way. As before, if we denote the initial velocity and the final velocity by v_i and v_f, the change in the kinetic energy, which must be given by the work done, is given as follows:

$$W = \Delta E_k = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}\frac{F\cos\theta}{a}(v_f^2 - v_i^2) = (F\cos\theta)\frac{v_f^2 - v_i^2}{2a} = (F\cos\theta)s \tag{7}$$

where in the second step, we used the fact that $a = (F\cos\theta)/m$. We indeed see that the work is given by (6).

In fact, this formula is satisfied not only when θ is an acute angle. Suppose θ is 180 degrees. In this case, you are exerting the force in the opposite direction the object is moving. Therefore, you decrease the speed of the moving object, which in turn implies that the kinetic energy decreases. In other words, the change of the kinetic energy is negative. This makes sense, since you did a negative work, as (6) is negative if θ is 180 degrees. Another such example is friction. If friction acts in the opposite direction an object moves, it does a negative work and therefore the kinetic energy of the object decreases.

Finally, let me conclude this article with two comments. First, those of you who know dot product will notice that (6) can be expressed using dot product. If you are curious about it, read my later articles listed in the section "Dot product." It is not that hard. Second, the unit for energy and work is denoted as "J" and pronounced as "Joule." For example, 1 J is 1 $\text{N} \cdot \text{m}$ which in turn is equal to 1 $\text{kg} \cdot \text{m}^2/\text{s}^2$.

Problem 1. You push a 4-kg box, initially at rest, with a horizontal force 5 N while it moves 10 meter. Assume that there is no friction between the box and the ground. After the pushing, what is the kinetic energy of the box? What is its speed?

Problem 2. A 4-kg box is initially moving with a speed 5 m/s. If 5 N friction is exerted on the box, how far will the box slide, before it stops?

Summary

- If you exert on an object a force F in the direction separated by angle θ from the moving direction, and move the object by the distance s. The work you have done is given by $W = Fs\cos\theta$.
- The kinetic energy of an object with mass m and speed v is given by $\frac{1}{2}mv^2$.
- If you do work on an object, the kinetic energy of the object increases by the work you have done.