Pressure of photon gas

In this article, we will calculate the pressure of photon gas given its energy. (The “gas” in photon gas means that there are multiple numbers of photons present.) In other words, we will obtain the photon gas version for Boyle’s law. To this end, we will take a similar step to one in our earlier article “Kinetic theory of gases.”

Let’s say that a photon’s momentum is given by p, and its velocity is given by $v_x\hat{\imath}+v_y\hat{\jmath}+v_z\hat{k}$. Of course, we naturally have $c^2=v_x^2+v_y^2+v_z^2$. Then what is the x-component of the momentum? Since it would be proportional to v_x, it is simply given by pv_x/c, and similarly for the y-component and the z-component. If you are not sure, solve Problem 1.

Problem 1. Check that $|\vec{p}|=p$ is satisfied if $\vec{p}$ is given by following.

$$\vec{p}=\frac{pv_x}{c}\hat{x}+\frac{pv_y}{c}\hat{y}+\frac{pv_z}{c}\hat{z}$$

Then (1) of “Kinetic theory of gases” becomes:

$$\Delta p=-\frac{2pv_x}{c} \tag{1}$$

(2) in that article becomes:

$$F=\frac{2pv_x/c}{2L/v_x}=\frac{pv_x^2/c}{L} \tag{2}$$

(3) in that article becomes:

$$F=\frac{N\bar{p}\bar{v_x^2}/c}{L} \tag{3}$$

(4) in that article becomes:

$$P=\frac{F}{L^2}=\frac{N\bar{p}\bar{v_x^2}/c}{L^3}=\frac{N\bar{p}\bar{v_x^2}/c}{V} \tag{4}$$

(5) in that article is the same:

$$\bar{v_x^2}=\bar{v_y^2}=\bar{v_z^2} \tag{5}$$

(6) in that article becomes:

$$c^2=\bar{v_x^2}+\bar{v_y^2}+\bar{v_z^2}=3\bar{v_x^2} \tag{6}$$

Plugging this into (4), we get:

$$P=\frac{N\bar{p}c}{3V} \tag{7}$$

Now, notice that $\bar{p}c$ is the average energy of a single photon. Therefore, $N\bar{p}c$ is the energy of all the photons in the gas. If we denote it by U, we have:

$$PV=\frac{U}{3} \tag{8}$$

In terms of the energy density $\rho \equiv U/V$, we have:

$$P = \frac{\rho}{3} \tag{9}$$

This is our conclusion.

Summary

- For photon gas, $PV = \dfrac{U}{3}$ (i.e. $P = \dfrac{\rho}{3}$.)