Physics solutions for mathematical problems

Theoretical physics is written in the language of mathematics. Without using mathematics, no theoretical physicist can do research. Therefore, the development of the relevant branches of mathematics is crucial for the development of theoretical physics. It gives more tools for theoretical physicists, and allows us to perform research in a new world, which has been hitherto inaccessible due to lack of the tool or the language.

All this is not surprising. It is a common sense that mathematics is helpful for physics. But, can physics be helpful for math?

Certainly, Newton invented calculus to use it for physics. If he didn't need to do a research on physics, he might have well not invented calculus. In that respect, physics was helpful for math. Physics problems give us motivation to investigate new mathematics. However, physics can be helpful for math more than that. Please read the following quote.

> Stereotypically, a physicist supposedly comes up with a mathematical way of phrasing a physics problem, and a mathematician helps solve the problem. The result is answers for the physicist, and interesting areas of research for the mathematician. For Seiberg–Witten theory, however, the situation was reversed. Mathematicians were looking to the physicists for answers to mathematical questions.[1]

Here, we see that a physics theory called "Seiberg-Witten theory" gives answers to mathematical questions. Indeed, physics can be used to "solve" mathematical problems. I put the quotation marks here because mathematicians regard mathematical problems as solved only when they are proved so "rigorously." i.e. they are logically shown that there is absolutely no possibility that the mathematical statements there are false. Physics solutions assume the laws of physics, which are based on experiments, and therefore cannot be regarded as rigorous by mathematicians. Nevertheless, it's still good to have a "physics proof" since it is, more often that not, as beautiful as mathematical proofs. Sometimes, a physics proof can be very valuable if the mathematical proof is not known. Moreover, ideas in physics proof can make a lot of motivations and spinoffs in mathematics. String theory, a branch of physics, is the best example. The most renowned string theorist, Edward Witten went on to win the Fields Medal, Nobel prize equivalent of mathematics.

Of course, it would be too hard to provide some of the examples in which string theory "solved" mathematical problems. Nevertheless, I recently devised a simple freshman physics proof of

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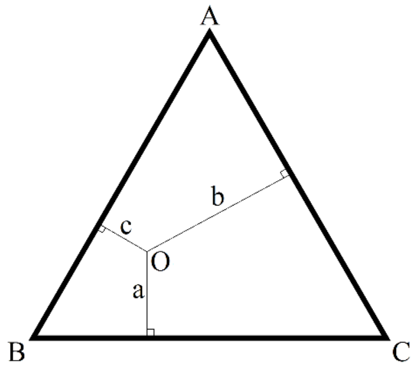
[1] What do Topologists want from Seiberg--Witten theory? (A review of four-dimensional topology for physicists) (http://arxiv.org/abs/hep-th/0207271)

elementary geometry problem. Of course, I doubt that I am the first one who devised this proof, but I came upon this example myself when I was trying to make some simple exercises for one of my review articles on physics. I proved that the sum of the distances from any point to the three sides of an equilateral triangle is constant. See the figure below. In other words, no matter where "O" is, "a+b+c" is constant.



I proved it in the following way. Assume the figure above is a triangular table, of which the legs are located at the three vertices, i.e. A, B, and C. What are the forces exerted on each leg if we place a heavy object with weight M kg at the point O, if we can neglect the mass of the table itself? I found out that the condition that the sum of the three forces must be equal to M kg gives out precisely the condition that "a+b+c" is constant. You will be invited to prove this theorem in our later article "Equilibrium of forces."