Root, cube root and n^{th} root

Let's say if you multiply the same number twice you get 9. What is the original number? The answer can be 3 or -3, since

$$3 \times 3 = (-3) \times (-3) = 9 \tag{1}$$

In such a case, we express:

$$3 = \sqrt{9} \tag{2}$$

We pronounce $\sqrt{9}$ as "root 9" or "square root of 9." Of course -3 can be the answer to our previous question, but as a root cannot result in multiple values, we only use the non-negative value as the answer. (Imagine if this were not the case. Then, if you try to calculate root 9 by pressing buttons on a calculator, you would get 3 and -3 as answers. This doesn't make much sense, as a calculator cannot spit out two answers on the screen. This concept of taking a single value instead of multiple values will be clear when we talk about the concept of functions later.)

In other words, if x is a non-negative number that satisfies $x^2 = y$, we have:

$$x = \sqrt{y} \tag{3}$$

Problem 1. Evaluate the following:

$$\sqrt{0}, \qquad \sqrt{9}, \qquad \sqrt{100}, \qquad \sqrt{49}, \qquad \sqrt{\frac{1}{4}}$$

In most cases, you have to resort to a calculator to calculate a root. For example, if you calculate $\sqrt{2}$ you will find:

$$\sqrt{2} = 1.414213562\cdots \tag{4}$$

On the other hand, the examples in Problem 1 were carefully chosen so that you don't need to use a calculator to find the answer.

Now, let me introduce the cube root. If $x^3 = y$, then $x = \sqrt[3]{y}$. For example, as $2^3 = 8$, we have $2 = \sqrt[3]{8}$. Notice that this is *not* equal to $3\sqrt{8}$, which means 3 multiplied by $\sqrt{8}$. The 3 in a cube root is written small.

There is also another big difference between the root and the cube root. Notice that the root of a negative number doesn't exist; if you multiply the same number twice you get always a non-negative number. (Remember if you multiply -3 by -3 you get 9 not -9.) On the other hand, the cube root of a negative number exists. For example, $\sqrt[3]{-27} = -3$ as $(-3)^3 = -27$.

We can actually generalize the square root and the cube root to n^{th} root. For example, if $x^n = y$ is satisfied, we have $x = \sqrt[n]{y}$.

Problem 2. Evaluate the following.

$$\sqrt[3]{27} = ? \qquad \sqrt[5]{-1} = ? \qquad 3\sqrt{16} = ? \qquad \sqrt{3\sqrt{4}+3} = ?$$