Similarity of triangle

There are simple ways to check whether two triangles are similar.

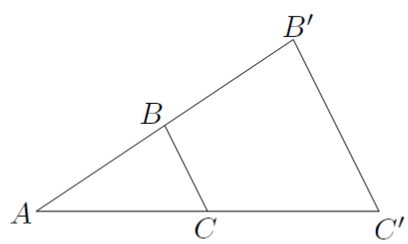
First, SSS (Side-Side-Side). All the corresponding sides have the same ratio. In other words, $\triangle ABC$ and $\triangle A'B'C'$ are similar if

$$\frac{\overline{AB}}{\overline{A'B'}} = \frac{\overline{BC}}{\overline{B'C'}} = \frac{\overline{CA}}{\overline{C'A'}} \tag{1}$$

Or, equivalently

$$\overline{AB} : \overline{BC} : \overline{AC} = \overline{A'B'} : \overline{B'C'} : \overline{A'C'} \tag{2}$$

Think it along this way. If you just change the overall size, without changing the shape, the ratios of each side must not change. For example, if you double the length of each side of the triangle, the shape won't change. See the figure.



$\triangle ABC$ and $\triangle AB'C'$ are similar, and (1) is satisfied as follows:

$$\frac{\overline{AB}}{\overline{AB'}} = \frac{\overline{BC}}{\overline{B'C'}} = \frac{\overline{CA}}{\overline{C'A}} = \frac{1}{2} \tag{3}$$

Second, SAS (Side-Angle-Side). If the ratio of the two sides of $\triangle ABC$ and the angle between them are same as the ratio of the two sides of $\triangle A'B'C'$ and the angle between them, $\triangle ABC$ and $\triangle A'B'C'$ are similar.

Third, AA (Angle-Angle). If the two angles of $\triangle ABC$ are same as the two angles of $\triangle A'B'C'$, then $\triangle ABC$ and $\triangle A'B'C'$ are similar. First, notice that this implies that the three angles of $\triangle ABC$ are same as the three angles of $\triangle A'B'C'$. This is easy to see as the sum of all the angles in a triangle is 180°, the other angle is determined once the two angles are determined. For example, if $\angle A = \angle A' = 40°$ and $\angle B = \angle B' = 60°$, then we are guaranteed that $\angle C = \angle C'$ as

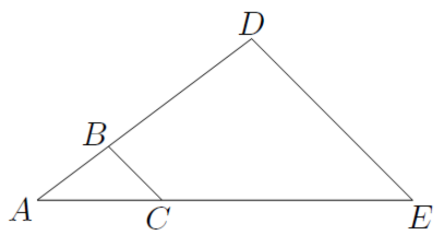
$$\angle C = 180° - \angle A - \angle B = 80° \tag{4}$$

$$\angle C' = 180° - \angle A' - \angle B' \tag{5}$$

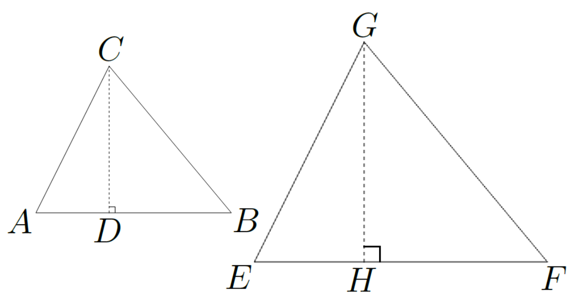
$$= 180° - \angle A - \angle B = 80° \tag{6}$$

Notice now that two triangles have to be similar, if the corresponding three angles are same, as this implies that the two triangles have the same shape.

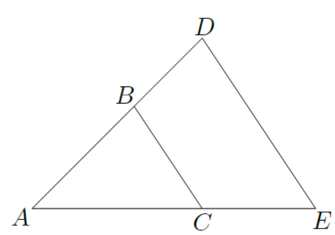
Problem 1. See the figure below. If $\overline{AC} = 7$, $\overline{CE} = 14$, $\overline{AB} = 5$, and $\angle ACB = \angle AED$, find $\overline{BD}$! (Hint[1])



Problem 2. See the figure below. If $\angle A = \angle E$, $\angle B = \angle F$, $\overline{AB} = 8$, $\overline{EF} = 12$ and the height of $\triangle ABC$ (i.e. $\overline{CD}$) is 6, what is the area of $\triangle EFG$? (Hint[2])



Problem 3. See the figure below. If $\overline{AC} = 3x$, $\overline{CE} = 2x$, $\overline{AB} = 3y$, $\overline{BD} = 2y$, $\overline{DE} = z$, find $\overline{BC}$. (Hint[3]) If the area of $\triangle ABC$ is 4, what is the area of $\triangle ADE$?

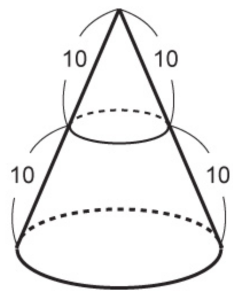


Problem 4. You see two right circular cones in the figure below. The smaller one is inside the bigger one. The bigger one has a slant height 20, and the smaller one has a slant height 10. How many times is the area of the base of the bigger cone as big as the area of the base of the smaller one?

[1] Use AAA similarity.

[2] Find $\overline{HG}$.

[3] Use SAS similarity.



Summary

- Two triangles are similar, if one of the three following criteria is satisfied:
- SSS: All the corresponding sides have the same ratio.
- SAS: The ratio of the two sides of a triangle and the angle between them are same as the ratio of the two sides of the other triangle and the angle between them.
- AA: The two angles of a triangle are same as the two corresponding angles of the other triangle.