The superiority of Hindu-Arabic numeral system

How many symbols do we need to have to express a number? We need ten, namely: 0, 1, 2, 3, $\cdots$, 8, 9. Using these symbols, we can express any number no matter how big is. For example, "two hundred fifty million five hundred sixty two thousand" can be represented as "250,562,000." That any number can be represented by only ten symbols can be too easily taken for granted, but the matter is not that simple. Consider when you actually utter a number in any European languages, you need a new word for 100 (which in English is a "hundred"), another new word for 1000 (which in English is a "thousand"), another new word for 1000,000 (which in English is a "million"), another new word for 1,000,000,000 (which in English is a "billion"). For bigger numbers, you always need more new names. Thus, it seems that the superiority of using the symbols 0, 1, 2, 3, ... is proven. This system of writing numbers, now called "Hindu-Arabic numeral system" was first invented in India about 1500 years ago, then transmitted to Arabia, then to Europe.

However, the superiority of Hindu-Arabic numeral system is not only in its simplicity of expressing numbers but also in its easiness of arithmetic. Nowadays, every elementary school kid can calculate 37×14. But, it was not always so. According to *Number, the language of science*, a book published by Tobias Dantzig in the 20th century, a German merchant of the 15th century asked a professor regarding where he should send his son for education. The professor answered that the son of the merchant could study at a German university, if learning addition and subtraction were sufficient, but he should study in Italy, if he wanted to learn multiplication and division.

So, why was multiplication so difficult? Because, the multiplication method then was difficult. For example, we now calculate 37×14 as

$$\begin{array}{r} 37 \\ \times 14 \\ \hline 148 \\ 37 \\ \hline 518 \end{array} \tag{1}$$

However, in the 13th century, they calculated as follows. In a modern nota-

tion,

$$\begin{aligned} 14 &= 8 + 4 + 2 \\ 37 \times 2 &= 74 \\ 37 \times 4 = 74 \times 2 &= 148 \\ 37 \times 8 = 148 \times 2 &= 296 \\ 74 + 148 + 296 &= 518 \end{aligned} \tag{2}$$

You see that they used doubling to calculate multiplication. For division, they used halving.

Notice that the use of Hindu-Arabic numeral system, which uses digits to express a number, was crucial in the calculation of (1). There you see that "37" is right below "14" in 148, which enables you to add the digits side by side. You wouldn't be able to do so, if you used other numeral system such as Roman numeral system or Chinese numeral system, which use new symbols for bigger numbers. In Hindu-Arabic numeral system, the same symbol can mean different numbers depending on its position. For example, "2" in 234 means two hundreds, while "2" in 327 means twenty.

I also want to note that the use of zero is crucial in Hindu-Arabic system. To see this, let's say that we didn't have the symbol for zero, because zero means that there is nothing. Then, two hundred thirty would be represented as 23, while two hundred three and would be represented as 23 as well! Actually, if you represent the number two hundred three (203) on an abacus, it would seem something like this.

You see that the digit for tens has to be empty. This corresponds to 0 in 203. The Indian word for zero was *sunya*, which means *empty*, but has no connotation of "nothing."

Thus, the Indian discovery of the number zero, was one of the most important discoveries in history of math, as it was crucial in Hindu-Arabic numeral system.

However, it seems that the modern method of multiplication was known in Europe long before the 13th century. Gerbert of Aurillac who later became the Pope Sylvester II in 999 used this method in his abacus. His method was explained in a book written by his student Bernelin. He used tokens carved with Hindu-Arabic numerals. However, there was no number zero, but instead just a empty digit in abacus. I do not know why this method by Gerbert had not been immediately universally used in Europe, despite his status as a pope. Perhaps, it's because Hindu-Arabic numeral

which is essential for this method was not popular in Europe. It was not until two hundred years later that Fibonacci promoted the use of Hindu-Arabic numeral system.